MATHEMATICS AND COMPUTER SCIENCE,

UNDERGRADUATE STUDY PROGRAMME, FIRST BOLOGNA CYCLE

COURSE DESCRIPTIONS

COMPULSORY COURSES

COMPULSORY COURSES FOR THE 1ST YEAR OF STUDY

Course name: **ALGEBRA I – MATRIX CALCULUS**
Number of ECTS credits: **6**

Content:
- Vectors, analytic geometry in space.
- Matrices. Types of matrices and basic operations with matrices. Rank of a matrix. Inverse.
- Systems of linear equations. Matrix interpretation and the solvability theorem. Elementary matrices, Gauss method. Determinants. Cramer's rule.

Course name: **ANALYSIS I – FOUNDATIONS OF ANALYSIS**
Number of ECTS credits: **6**

Content:
- Natural numbers. Rational numbers. Real numbers. Complex numbers.
- Sequences of real numbers. Limits and accumulation points. Cauchy condition. Upper and lower limit. Monotone sequences. Bolzano-Weierstrass theorem.
- Series. The convergence criteria. Absolutely and conditionally convergent series.
- Functions of real variables, even and odd functions, periodicity. Limits of functions, left and right limits. Continuity. Continuous functions on closed intervals limited. Bisection method for finding zeros.
- Elementary functions. Cyclometric functions.

Course name: **DISCRETE MATHEMATICS – SET THEORY**
Number of ECTS credits: **6**

Content:
- Fundamentals of mathematical theory, propositional calculus, truth tables, predicate calculus.
- Formal languages.
- Basic concepts of mathematical logic.
- Ways of describing sets. Basic relations between sets, basic operations on sets or families of sets. Power set. Relations. Graphs. Equivalence relations. Partial and linear ordering. Lattices and Boolean algebra. Well-ordered sets. Functions. Special types of functions. Categories.
- Finite and infinite, countable and uncountable sets.
- Cardinal and ordinal numbers. Peano arithmetics, mathematical induction.
- NBG and ZFC systems of axioms of set theory. Axiom of choice. Zorn's lemma.
- Fundamentals of symbolic computation (Mathematica).

Course name: **COMPUTER PRACTICUM**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
The faculty network and basic usage rules:
- Use of the faculty computer network, login methods, password changing procedure, e-mail and mailing list usage, access to e-materials.

OS Linux basics:
- Use of the Linux OS and its many flavors, BASH shell usage basics.

Programming language C:
- Use of the C programming language and solving simple problems with it.

Course name: **PROGRAMMING I**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
- Basic building blocks of a computer program.
- Basic syntax of the programming language Java: Variables, types and expressions. Basic I/O operations. Decision statements. Control structures. Functions and parameters. Programs. Structural decomposition
- Basic data structures: Simple types. Arrays. Records. Characters and strings. Data representation in computer memory. Memory allocation. Linked structures. Stack. Queue. List. Tree.
- Algorithms and problem solving: What is an algorithm? Problem solving strategies. The role of algorithms in problem solving. Algorithm implementation strategies. Debugging. Recursion – recursive functions, divide-and-conquer principle, backtracking, implementation of recursion.
- Programming languages overview: Types of programming languages. Flow control. Functions. Subprograms. Namespaces.
- Declarations and types: Types. Declarations of types. Safe typing. Type checking. Subtypes. Classes. Polymorphism.
- Abstraction mechanisms: Data abstractions. Simple types. Composite types. Flow abstractions. Subprograms and functions. Abstract data types. Objects and classes. Patterns. Modules.

Course name: **ALGEBRA II – LINEAR ALGEBRA**
Number of ECTS credits: **6**

Content:
- Groups, rings,domains. Polynomial rings.
- Vector space. Subspaces, linear operators. Linear independence. Basis and the dimension vector space.
- Eigenvalues. The characteristic and minimal polynomial.
- Inner product. Orthogonal systems. Gramm-Schmidt orthogonalization process. Norm. Norm of the matrix and the operator. Normal and related operators.
- Convexity in the vector space.
- Normed vector spaces as metric spaces. Isometries of R2 and R3.

Course name: **ANALYSIS II – INFINITESIMAL CALCULUS**
Number of ECTS credits: **6**

Content:
- Derivative. Mean value theorems. Differentiation of monotone functions. L'Hopital's rule. Higher derivatives. Taylor's formula. Local extrema. Convex and concave functions. Inflection points. The tangent method of finding the zeros.

- Indefinite Integral. Definite Integrals. Darboux and Riemann sums. Leibniz-Newton formula. Mean value theorems. Integration methods. Application of the definite integral in geometry. Improper integral. Numerical integration.
- The logarithm, the number e and the definition of exponentiation with the real exponent.
- Drawing planar curves.
- Sequences and function series. Power series. Taylor series. Elementary complex functions.

Course name: **DISCRETE MATHEMATICS II – COMBINATORICS**
Number of ECTS credits: **6**

Content:
1) Mandatory topics (approximately 70%-75% of contact hours):

- Introduction. What is combinatorics? Sample problems (such as: derangements, Euler's officers, Kirkman's schoolgirls, a Ramsey game etc.).
- Basic Combinatorial Principles. The addition principle. The double counting principle. The multiplication principle. The equality principle. The pigeonhole principle (Dirichlet's principle). Generalized Pigeonhole Principle.
- Elementary Combinatorics. Ordered selections with repetition. Ordered selections without repetition (special case: permutations). Unordered selections without repetition, formula for binomial coefficients. Quotient representation of binomial coefficients. Unordered selections with repetition. Permutations with repetition. Pascal's identity and Pascal's triangle. Binomial Theorem and consequences. Principle of Inclusion-Exclusion. Counting derangements. Counting surjective functions.
- Recurrence Relations. Examples of recurrence relations. Fibonacci numbers. Linear recurrence relations with constant coefficients. Derangements, revisited.
- Distributions. Distributing labeled elements into unlabeled cells. Stirling numbers of the second kind, recurrence relation and relation to Bell numbers. Distributing labeled elements into labeled cells. Distributing unlabeled elements into unlabeled cells. Partitions of a positive integer n of order k. Distributing unlabeled elements into labeled cells.
- Graphs. Basic definitions. Graph isomorphism. Walks, trails, paths, connected components. Trees and forests: definitions and basic properties. Spanning trees. Minimum Spanning Trees: Kruskal's Algorithm. Eulerian Graphs: Euler's Theorem. Hamiltonian graphs. Hall's Theorem, systems of distinct representatives. Application: Latin squares. Graph vertex- and edge colorings. Planar graphs.

2) For the remaining 25%-30% of contact hours, the instructor shall, at his / her discretion, select topics from the following list:

2.1) Additional topics from above chapters:
- Recurrence Relations. Formal power series. Generalized Binomial Theorem. Catalan and Bell numbers.
- Graphs. Cayley's Theorem. The Traveling Salesman Problem. Digraphs. Shortest paths, Dijkstra's algorithm. Tournaments. Kőnig-Egérvary's theorem. Menger's theorem. Chromatic polynomial of a graph. Graph homomorphisms. Ramsey's theorem.

2.2) Topics from additional chapter:
- Introduction to Designs. Balanced Incomplete Block Designs - definition and examples. Steiner triple systems. Incidence matrix of a design. Fisher's inequality. Symmetric designs. Resolvable Designs. Solution to Kirkman's schoolgirl problem. Finite projective planes, affine planes. Difference methods. Hadamard designs and Hadamard matrices.

Course name: **MATHEMATICAL PRACTICUM**
Number of ECTS credits: **6**

Content:
- Presentation software (eg PowerPoint), spreadsheets (eg Excel)
- Text editors (eg WinEdt, TextPad, Emacs, Auctech, Open Office, ...)
- Basics of TeX and LaTeX (MikTeX, TeTex, GSview, Acrobat Reader, ...)
- The basic image tools (pdf, eps), working with image formats, including images in LaTeX, scanning and the use of digital cameras.

Course name: **COMPUTER SYSTEMS**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
Computers:
- Computer Function and Interconnection
- The Memory: Cache, Internal, External
- Input/Output
- Processor: Structure, Arithmetic, Instruction Set
- Parallel, Distributed and Multicore Processing

Operating Systems:
- Structures
- Processes, Threads & Concurrency
- Interprocess Communication
- Memory Management: Main Memory, Virtual Memory
- File-System and Protection

Networks:
- Network Layers and Reference Models
- Network layer: Service, Design Issues, Routing, Internet, DNS
- Transport layer: Service, UDP, TCP, TLS
- Application layer: The World Wide Web

Security:
- Cryptography, Symmetric-Key, Public-Key
- Digital Signatures
- Authentication

COMPULSORY COURSES FOR THE 2ND YEAR OF STUDY

Course name: **ALGEBRA III – ABSTRACT ALGEBRA**
Number of ECTS credits: **6**

Content:
- Introduction to number theory, Euclidean algorithm, congruences.
- Polynomials of one variable. Euclidean algorithm. Zeros of polynomials. Solving algebraic equations. Polynomials of several variables. Symmetric polynomials. Fundamental theorem of algebra.
- Grupoids, semigroups and groups. Homomorphisms of groups. The normal subgroups and factor groups. Group families. Groups given by generators and relations. Sylow theorems.

Course name: **ANALYSIS III – FUNCTIONS OF MANY VARIABLES**
Number of ECTS credits: **6**

Content:

- Metric spaces. Cauchy-Schwarz inequality. Open and closed sets. Compactness and connectedness. Sequences in metric spaces. Cauchy sequences and complete spaces. Continuity and uniform continuity. Properties of continuous mappings.
- Functions of several variables. Continuity, partial differentiability. Differential. Mappings from Rn to Rm. Jacobian matrix. Chain rule.
- Higher order partial derivatives. Taylor's formula. Theorem on the inverse and implicit functions. Local extrema and constrained local extrema
- Double and Multiple integrals. Properties. Theorems on existence. The substitution of new variables. Calculation and application.
- Proper and improper integrals with the parameter.

Course name: **DATA STRUCTURES AND ALGORITHMS**
Number of ECTS credits: **6**

Content:

The basic mathematical tool

- The evaluation function O, □, □ and the differences between them
- What is the complexity of the problem and what is the complexity of solutions
- probability and randomness
- computational models: Pointer Machine, RAM, parallel computer

Basic data structures

- implicit data structures: array, stack, queue, heap
- explicit data structures: linked list, tree
- problems with concurrency

The basic abstract types and their performance

- Dictionary: hash tables, AVL, red-black, B – trees
- priority queue: heap, VEB
- ranking and selection

Sorting and friends:

- selection sort, merge sort, quicksort
- sorting in linear time
- sorting on parallel machines
- median and the kth element

Basic algorithmic techniques

- greedy algorithms
- divide and conquer
- dynamic programming

Algorithms on graphs and networks

- search in width and in depth
- connected components
- minimum spanning trees
- shortest paths in graphs
- network flow
- parallel algorithms on networks with a view of the Internet

Selected algorithms

- mathematical algorithms with a view on cryptography: matrix multiplication, solving systems of equations, FFT, finding the greatest common divisor, modular arithmetic, exponents
- algorithms on strings with a view on bioinformatics: finding a substring
- for all problems we shall look at some basic parallel solutions

Course name: **INTRODUCTION TO NUMERICAL CALCULATIONS**
Number of ECTS credits: **6**

Content:
- Fundamentals of numerical computa-tion. Floating point computation and numerical error. Stable computational processes and the sensitivity. The total error.
- Nonlinear equations. Bisection. Iteration methods. Tangent method, secant method. Algebraic equations. Systems of nonlinear equations. Newton's method.
- Systems of linear equations. LU and Cholesky decompositions. Gauss elimination. Diagonally dominant and tridiagonal matrices. Condition number.
- Solving linear systems iteratively.
- Least squares problem. Predetermined systems. QR decomposition. SVD decomposition.
- Eigenvalues of matrices. Power method and inverse power method. Gerschgorin's and Schur's theorem.
- Interpolation of functions. Polynomial interpolation. Divided differences. Interpolation with splines.
- Numerical derivation. Numerical integration. Newton-Cotes rules. Composed rules. Gauss quadratures. Euler-Maclaurin formula and Romberg extrapolation. Multivariate integration and cubature rules.
- Bezier curves. De Casteljau algorithm. Bernstein polynomials. Properties of Bezier curves. Subdivision. Degree elevation.

Course name: **INTRODUCTION TO DATABASE SYSTEMS**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
- Introduction
- Entity-Relationship model
- Relational model
- Relational algebra and calculus
- SQL and QBE
- Disks and files
- Indexes
- Evaluation of relational operations
- Query optimization
- Transactions
- Concurrency control and crash recovery
- Database design

Course name: **ALGEBRA IV – ALGEBRAIC STRUCTURES**
Number of ECTS credits: **6**

Content:
- Rings. Ideals. Homomorphism of rings. Factor rings. Integral domain. Euclidean rings. Principal ideal rings. Gaussian coils. Gaussian integers. Chinese remainder theorem.
- Fields. Subfields. Extensions. Final extension.
- The rate of extension. The degree theorem. Simple algebraic extension. Splitting field.
- Constructions with ruler and compass. Squaring the circle. Angle trisection. Doubling Cube. Constructions of regular polygons.

Course name: **PROBABILITY**
Number of ECTS credits: **6**

Content:
Basic combinatorics
- Rule of product, rule of sum.
- Variations and variations with repetition.
- Combinations and combinations with repetition.
- Permutations and permutations with repetition.
- Binomial formula with its generalizations.

Outcomes and events
- Sample space, events, probability measure.
- Computations with events.
- Conditional probability and independence.

Random variables
- Random variables and their distributions.
- Basic discrete distributions.
- Continuous random variables.

Multivariate distributions
- Definition of discrete multivariate distributions.
- Discrete multivariate distributions.
- Multivariate continuous distributions.
- Conditional distributions and independent random variables.

Expectation and variance
- Expectation and its properties
- Variance and covariance
- Conditional expectation

Generating functions
- Definition and examples.
- Branching processes.

Approximation of distributions
- Convergence in Distribution.
- Normal approximation of a sum of random variables.
- Poisson's approximation.

Course name: **FORMAL LANGUAGES AND COMPUTABILITY**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
Finite automata and regular expressions:
- Computation model, finite automaton – DFA and NFA
- Alphabet, language, regular expression/language
- The relation between DFA, NFA and regular expressions
- Using FA to solve problems
- RE pumping lemma, non-regular languages

Grammars, context (non)free languages and stack automata:
- Grammar, derivation tree, attribute grammar
- Context-dependent and context-free languages, stack automaton
- Grammar normal forms: Greibach, Chomsky (CNF)
- Transforming grammars into CNF
- CYK algorithm
- Pumping lemma for CFG
- Operations on languages (intersection, union …)

Turing machines and their languages:
- Turing machine and its derivations, RAM
- Church-Turing thesis
- Recursively enumerable languages, Chomsky hierarchy
- Unsolvable and undecidable problems, the stopping problem, Rice theorem and PCP

Introduction to the theory of complexity:
- P and NP – relation between them
- Problem translation, NP completeness
- NP complete problems

Course name: **PROGRAMMING II**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
- Introduction
- Lambda calculus
- Syntax
- Basic structures
- Functional languages
- Imperative languages
- Types
- Modules
- Objects and classes

Course name: **CODING THEORY**
Number of ECTS credits: **6**

Content:
- Mathematical background (groups, rings, ideals, vector spaces, finite fields)
- Basic concepts in coding theory
- Algebraic methods for the construction of error correcting codes
- Hamming codes
- Linear codes
- Binary Golay codes
- Cyclic codes
- BCH codes
- Reed-Solomon codes
- Bounds (Hamming, Singleton, Johnson's bound , ...)

COMPULSORY COURSES FOR THE 3RD YEAR OF STUDY

Course name: **GRAPH THEORY**
Number of ECTS credits: **6**

Content:
- Fundamental concepts. What is a graph? Paths, cycles, and trails. Vertex degrees and counting. Directed graphs.
- Trees and distances. Basic properties. Spanning trees and enumeration. Trees and optimization.
- Matchings and factors. Matchings and covers. Algorithms and applications. Matchings in general graphs.
- Connectivity and paths. Cuts and connectivity. k-connected graphs. Flows in networks.
- Coloring of graphs. Vertex colorings and upper bounds. Chordal graphs and perfect graphs.
- Edges and cycles. Line graphs and edge colorings. Hamiltonian cycles.

- Planar graphs. Embeddings and Euler's formula. Characterizations of planar graphs.

Course name: **STATISTICS**
Number of ECTS credits: **6**

Content:
Sampling:
- The concept of random sampling
- Sampling distribution and standard error
- Examples of sampling and their standard errors
- Stratified sampling and examples of allocations

Parameter estimation:
- The concept of a statistical model
- Parameter space, estimators, sampling distribution
- Maximum likelihood method
- Asymptotic properties of the maximum likelihood method
- Rao-Cramér inequality, optimality of estimates, factorization theorem

Hypothesis testing:
- Problem formulation
- Statistical tests, test size, power of tests
- Examples of statistical tests
- Wilks' Theorem
- Neyman-Pearson lemma, theory of optimality

Linear models:
- Assumptions of linear models and examples
- Parameter estimation
- Gauss-Markov theorem
- Generalizations of linear models
- Applications

Course name: **PROGRAMMING III – CONCURRENT PROGRAMMING**
Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:
- Concurrent programming concepts; Techniques for parallelizing programs
- Synchronization, atomic actions.
- Critical sections: locks
- Parallel programming: barriers, barrier synchronization, bag of tasks paradigm
- Semaphores: basic concepts and uses, the method of passing the baton
- Introduction to Pthreads library
- Monitors and conditional variables: basic concepts, synchronization techniques, programming
- Distributed memory programming:
- message passing, remote procedure call, remote method invocation
- Introduction to MPI
- Examples of multithreaded, parallel and distributed programming.
- Map-Reduce model

Course name: **INTRODUCTION TO MACHINE LEARNING AND DATA MINING**
Number of ECTS credits: **6**

Content:
Machine learning techniques:
- From basic to more advanced machine learning methods such as: decision trees, decision rules, regression, support vector machines, association rules …
- The use of these techniques on real-life problems
- Building and evaluation of prediction and description models from the data.

Data mining:
- The use of machine learning techniques in the frame of the data mining process.
- Description of the CRISP methodology – data mining as a process consisting of the following steps: problem understanding, data understanding, data preparation, modeling, evaluation and deployment.
- The use of existing, open source data mining tools (e.g.: WEKA, Orange, A-Priori …)

Course name: **CRYPTOGRAPHY AND COMPUTER SAFETY**
Number of ECTS credits: **6**

Content:
- Classical ciphers and hoistorical development
- Fiestel's cipher and AES (Advanced Encryption Standard)
- Finite fields and Extended Euclidean algorithm
- Public crypto systems, one-way functions and related problems from number theory (testing primality, factorization of integers, discrete logarithm problem)
- Hash functions and message integrity (authentication)
- Key exchange protocols and identification protocols
- Pseudo random number generator
- Other protocols (flipping a coin over the telephone, mental poker, secret sharing, verification codes, vizual cryptography, zero knowledge proofs)
- Public key infrastructure (PKI), certificate authority (CA)
- Broader view on cryptography – security of information and network security

Course name: **MATHEMATICAL MODELLING**
Number of ECTS credits: **6**

Content:
- Introduction. What is mathematical modeling? The role of mathematical models in natural sciences and economics. Types of mathematical models.
- Programming tools. A short overview of Octave/Scilab.
- Optimization. Critical point, minimum, maximum, saddle. Taylor's formula for scalar fields. Local extrema and local extrema under constraints. Newton's method. Applications: discrete catenary, truss stability etc.
- Calculus of variations. Standard problem of variation calculus. Isoperimetric problems. Applications: catenary, brachistochrone, truss oscillations, etc.
- Linear programming. What is a linear program? Examples of linear programs: optimal diet, flow in a network etc. Forms of linear programs. The fundamental theorem of linear programming. Simplex method. Duality. Integer linear programming and LP relaxation. Applications.
- Differential equations and systems of differential equations as mathematical models in natural sciences. Motivational examples. Equilibrium. (Linear) Stability of equilibria. Phase portraits. The basics of Poincare-Bendixon theory. The basics of bifurcation theory. Applications: epidemic models, models of competition, models of symbiosis, predator-prey dynamics, molecular kinetics, basic neurological models, models in economics.

Course name: **OPTIMIZATION METHODS**
Number of ECTS credits: **6**

Content:
Topics denoted by * are optional.
Linear programming:
- Brief introduction to convexity and polyhedra.
- Examples for LP problems, model setup, geometric interpretation.
- Simplex method, revised simplex method, two-phase simplex method.
- Duality (Farkas' lemma, hyperplane separation).
- Algorithmic issues: finiteness, cycling, optimality, LU-factorization.
- Matrix games*.
- Continuous knapsack*, Dantzig's solution*.
- Integer knapsack, dynamic programming solution.
- Cutting stock problems, column/row generation.
- Problem size, complexity of algorithms, polynomial-time algorithms.
- Ellipsoid method and the separation routine, the Grőtschel-Lovász-Schrijver theorem (with no proof).
- Network simplex*, transshipment* and transportation problems.*

Discrete optimization and complexity:
- Shortest paths (Formulation as LP and as min-cost
- flow problem, Dijkstra's algorithm, Bellman-Ford equations, connection with dynamic programming).
- Maximum flow algorithms (path and arc-flow formulations, augmenting paths, Ford-Fulkerson algorithm, max-flow min-cut theorem; link with LP duality; Dinitz, Edmonds-Karp improvements).
- Bipartite matching (unweighted case, integer linear programming formulation, max flow formulation, alternating path algorithm; weighted case, assignment problem, Hungarian algorithm).
- Spanning trees (greedy algorithm).
- Complexity (introduction to NP: decision problems, polynomial transformations, NP-completeness; satisfiability problem and Cook's theorem; some examples of polynomial transformations and NP-completeness proofs; “equivalence" of
- optimization and decision versions, solution of optimization problems by binary search).
- Approximation algorithms (concepts, vertex cover, spanning tree algorithm for the metric TSP (possibly including a discussion of Eulerian tours), proof that TSP without triangle inequalities cannot be approximated; next-fit for bin packing, set covering).

Other possible topics (to be selected at the discretion of the instructor):
- Non-bipartite matching (Tutte's and Lovász' matrix theorems, Edmonds' blossom algorithm, Edmonds-Gallai structure theorem, Chinese postman problems, planar max-cut and Christofides' TSP approximation algorithm).
- Integer programming (modeling, branch and bound, polyhedral approaches: cutting planes, valid inequalities)
- Scheduling problems (polynomial-time algorithms for one-machine scheduling: WSPT for
- $1 / r = 0 / \sum w_j c_j$, EDD for $1/ d / T_{max}$, LP-based
- 2-approximation algorithm for makespan minimization on unrelated parallel
- machines).
- Dynamic programming algorithms for hard problems (knapsack, traveling salesman problem)
- MAX-SAT (Johnson's approximation algorithm, proof via pseudo-Boolean optimization)
- Local search metaheuristics (main concepts: neighborhood structures, steepest descent – hill climbing, simulated annealing).
- Nonlinear programming (extremum of a function from Rn to R, gradient and the Hesse matrix, unconstrained minimization, gradient method, constrained minimization, transformation to the
- unconstrained problem, Karush-Kuhn-Tucker conditions).

ELECTIVE COURSES

Course name: **PRACTICAL TRAINING IN THE WORKPLACE**
Number of ECTS credits: **6**

Content:
Student, according to their own interests, selects or proposes a department in the organization where they would like to get a work experience placement. The content of work experience will thus depend mainly on the chosen field of work and student's activities. It is defined jointly by both student and mentor from the organization in the work experience plan, which is confirmed by study program coordinator.

Work experience plan includes description of a problem and goals of a project in which the student is about to participate as well as the students assignments.