# MATHEMATICS IN ECONOMICS AND FINANCE, UNDERGRADUATE STUDY PROGRAMME, FIRST BOLOGNA CYCLE

# COURSE DESCRIPTIONS (MF-17)

## COMPULSORY COURSES

## COMPULSORY COURSES FOR THE 1ST YEAR OF STUDY

Course name: **ALGEBRA I - MATRIX CALCULUS**
Number of ECTS credits: **6**

**Content:**
- Vectors, analytic geometry in space.
- Matrices. Types of matrices and basic operations with matrices. Rank of a matrix. Inverse. Systems of linear equations. Matrix interpretation and theorem of solvability. Elementary matrices, Gauss method. Determinants. Cramer's rule.

Course name: **ALGEBRA II - LINEAR ALGEBRA**
Number of ECTS credits: **6**

**Content:**
- Groups, rings, fields. Ring of polynomials.
- Vector space. Subspaces, linear operators. Linear independence. Basis and dimension of
- vector space.
- Eigenvalues. The characteristic and minimal polynomial.
- Inner product. Orthogonal systems. Gramm-Schmidt process of ortogonalization. Norm. Norm of the matrix and the operator. Normal and related operators.
- Convexity in the vector space.
- Normalized vector spaces as metric spaces. Isometries of R2 and R3.

Course name: **ANALYSIS I - FOUNDATIONS OF ANALYSIS**
Number of ECTS credits: **6**

**Content:**
- Natural numbers. Rational numbers. Real numbers. Complex numbers.
- Sequences of real numbers. Limits and accumulation points. Cauchy condition. Upper and lower limit. Monotone sequences. Bolzano-Weierstrass theorem.
- Series. The convergence criteria. Absolutely and conditionally convergent series.
- Functions of real variables, even and odd functions, periodicity. Limits of functions, left and right limits. Continuity. Continuous functions on closed intervals limited. Bisection method for finding zeros.
- Elementary functions. Cyclometric functions.

Course name: **ANALYSIS II – INFINITESIMAL CALCULUS**
Number of ECTS credits: **6**

**Content:**
- Derivative. Mean value theorems. Differentiation of monotone functions. L'Hopital's rule. Higher derivatives. Taylor's formula. Local extrema. Convex and concave functions. Inflection points. Tangent method of finding the zeros.

- Indefinite integral. Definite Integrals. Darboux and Riemann sums. Leibniz-Newton formula. Mean value theorems. Integration methods. Applications of the definite integral in geometry. Improper integral. Numerical integration.
- The logarithm, the number e, and the definition of exponentiation with the real exponent.
- Drawing planar curves.
- Sequences and function series. Power series. Taylor series. Elementary complex functions.

Course name: **DISCRETE MATHEMATICS I – SET THEORY**
Number of ECTS credits: **6**

**Content:**
- Fundamentals of mathematical theory, propositional calculus, truth tables, predicate calculus.
- Formal languages.
- Basic concepts of mathematical logic.
- Ways of describing sets. Basic relations between sets, basic operations on sets or families of sets. Power set. Relations. Graphs. Equivalence relations. Partial and linear ordering. Lattices and Boolean algebra. Well-ordered sets. Functions. Special types of functions. Categories.
- Finite and infinite, countable and uncountable sets.
- Cardinal and ordinal numbers. Peano arithmetics, mathematical induction.
- NBG and ZFC systems of axioms of set theory. Axiom of choice. Zorn's lemma.
- Fundamentals of symbolic computation (Mathematica).

Course name: **DISCRETE MATHEMATICS II - COMBINATORICS**
Number of ECTS credits: **6**

**Content:**
- The principle of the sum, product. Counting pairs. Elementary combinatorics. Assignment. Assignment within the set. The existence of a 1-factor. Assignment between two sets, Hall's theorem. König's theorem, applications. Recursion. Generating functions. Linear recursion with constant coefficients. Applications of combinatorics. Inclusion-exclusion principle. Rook polynomial. Möbius inversion. Partially ordered sets and the Möbius function. Theorem on the inversion. Designs. Finite projective planes. Correction code. Steiner systems. Kirkman schoolgirl problem. Ramsey theorem, proof and application. Polya Theory. Burnside's lemma. Polya's theorem.
- Graphs, examples of graphs. Trees. Basic properties, enumeration of trees. The cheapest tree. Operations on graphs. Product of graphs. Deck graphs and voltage graphs. Graphs and groups. Graph automorphism group. Cayley graphs and Frucht theorem. Symmetric graphs. Planarity and duality. Criterion of planarity. Graph embeddings in other plots. Duality and Euler's theorem. Graph coloring. Coloring vertices. Coloring edges. Chromatic polynomial. Directed graphs. Eulerian digraphs. Tournaments. Markov chains. Connectivity. Menger's and Hall's theorem. Different versions of Menger's theorem and Ford-Fulkerson's theorem. Matroid theory. Definitions. Matroids and graphs. Examples of matroids and applications

Course name: **MATHEMATICAL TOPICS IN ENGLISH I**
Number of ECTS credits: **6**

**Content:**
Lectures are given on the most current research topics in the field of mathematics, which may include the following topics:
- Algebra,
- Analysis,
- Discrete mathematics,
- Financial mathematics,
- Cryptography,
- Computer intensive methods and applications,
- Linear programming,
- Statistics.

Course name: **MATHEMATICAL PRACTICUM I**
Number of ECTS credits: **6**

**Content:**
- Programs for presentations (eg PowerPoint), spreadsheet (eg Excel)
- Text editors (eg WinEdt, TextPad, Emacs, Auctech, Open Office, ...)
- Introduction to TeX and LaTeX-a (MikTeX, tetex, GSview, Acrobat Reader, ...)
- The basic tools to produce images (pdf, eps), working with the formats of images including images in LaTeX
- Scanning and use of digital cameras

Course name: **COMPUTER SCIENCE I**
Number of ECTS credits: **6**

**Content:**
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:

- Basic building blocks of a computer program (using the syntax of the programming language Java):

Variables, types and expressions. Basic I/O operations. Decision statements. Control structures. Functions and parameters. Programs. Structural decomposition.

- Basic data structures:

Simple types. Arrays. Records. Characters and strings. Data representation in computer memory. Memory allocation. Linked structures. Stack. Queue. List. Tree.

- Algorithms and problem solving:

What is an algorithm? Problem solving strategies. The role of algoirithms in problem solving. Algorithm implementation strategies. Debugging. Recursion – recursive functions, divide-and-conquer principle, backtracking, implementation of recursion.

- Programming languages overview:

Types of programming languages. Flow control. Functions. Subprograms. Namespaces.

- Declarations and types:

Types. Declarations of types. Safe typing. Type checking. Subtypes. Classes. Polymorphism.

- Abstraction mechanisms:

Data abstractions. Simple types. Composite types. Flow abstractions. Subprograms and functions. Abstract data types. Objects and classes. Patterns. Modules.

Course name: **COMPUTER PRACTICUM**
Number of ECTS credits: **6**

**Content:**
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:

- Hardware.
- The structure of a computer system, the memory hierarchy, devices, bus, I/O devices, CPU. Operating system. Processes, synchronisation, devices, process scheduler, device managers.
- OS Linux basics.
- The Linux OS and its Slovenian flavour – Pingo Linux. Usage of the BASH shell.
- Programming languages.
- Programming language types. Imperative, object-oriented, logical, declarative programming languages. Concepts of programming languages. Iteration, data structures, control structures,

functions, subroutines. Basic data structures: fields, arrays and lists. The C programming language.
- The object oriented model.
- The concepts of the object oriented model: objects, classes, fields, methods, inheritance, polymorphism, multiple inheritance, interfaces and abstract classes. Abstract data types. Examples of abstract data types. The Java programming language.

## COMPULSORY COURSES FOR THE 2ND YEAR OF STUDY

Course name: **ANALYSIS III - FUNCTIONS OF MANY VARIABLES**
Number of ECTS credits: **6**

**Content:**
- Metric spaces. Cauchy-Schwarz inequality. Open and closed sets.
- Compactness and connectedness. Sequences in metric spaces. Cauchy sequences and complete metric spaces. Continuity and uniform continuity. Properties of continuous mappings.
- Functions of several variables. Continuity, partial differentiability. Differential mapping from Rn to Rm. Jacobian matrix. Chain rule of differentiation.
- Higher order partial derivatives. Taylor's formula. Theorem on the locally inverse function and on implicit functions. Local extremal problems, constrained extremal problems.
- Double and multiple integrals. Properties. The conditions on the existence. The introduction of new variables.
- Calculation and application.
- Proper and generalized integrals with parameter. Beta and Gamma functions. Stirling formula.

Course name: **ALGEBRA III - ABSTRACT ALGEBRA**
Number of ECTS credits: **6**

**Content:**
- Introduction to number theory, Euclidean algorithm, congruences.
- Polynomials o in single variable. Euclidean algorithm. Zeros of polynomials. Solving algebraic equations. Polynomials in several variables. Symmetric polynomials. Fundamental theorem of algebra.
- Grupoids, semigroups and groups. Homomorphisms of groups. Normal subgroups and factor groups. Families of groups. Groups given by generators and relations. Sylow theorems.

Course name: **PROBABILITY**
Number of ECTS credits: **6**

**Content:**
***Basic combinatorics***
-Rule of product, rule of sum.
-Variations and variations with repetition.
-Combinations and combinations with repetition.
-Permutations and permutations with repetition.
-Binomial formula with its generalizations.

***Outcomes and events***
-Sample space, events, probability measure.
-Computations with events.
-Conditional probability and independence.

***Random variables***
-Random variables and their distributions.
-Basic discrete distributions.
-Continuous random variables.

***Multivariate distributions***
-Definition of discrete multivariate distributions.
-Discrete multivariate distributions.
-Multivariate continuous distributions.
-Conditional distributions and independent random variables.

***Expectation and variance***
-Expectation and its properties
-Variance and covariance
- Conditional expectation

***Generating functions***
-Definition and examples.
-Branching processes.

***Approximation of distributions***
-Convergence in Distribution.
-Normal approximation of a sum of random variables.
-Poisson's approximation.

Course name: **INTRODUCTION TO NUMERICAL CALCULATIONS**
Number of ECTS credits: **6**

**Content:**
- Fundamentals of numerical computa-tion. Floating point computation and numerical error. Stable computational processes and the sensitivity. The total error.
- Nonlinear equations. Bisection. Iteration methods. Tangent method, secant method. Algebraic equations. Systems of nonlinear equations. Newton's method.
- Systems of linear equations. LU and Cholesky decompositions. Gauss elimination. Diagonally dominant and tridiagonal matrices. Condition number.
  Solving linear systems iteratively.
- Least squares problem. Predetermined systems. QR decomposition. SVD decomposition.
- Eigenvalues of matrices. Power method and inverse power method. Gerschgorin's and Schur's theorem.
- Interpolation of functions. Polynomial interpolation. Divided differences. Interpolation with splines.
- Numerical derivation. Numerical integration. Newton-Cotes rules. Composed rules. Gauss quadratures. Euler-Maclaurin formula and Romberg extrapolation. Multivariate integration and cubature rules.
- Bezier curves. De Casteljau algorithm. Bernstein polynomials. Properties of Bezier curves. Subdivision. Degree elevation.

Course name: **MICROECONOMIC ANALYSIS**
Number of ECTS credits: **6**

Preliminaries and economic intuition
- Market mechanism
- demand and supply

Consumer behavior
- Preferences, utility function
- Consumer choice
- Individual and aggregate demand
- Consumer surplus

Theory of the firm
- Production and production costs
- Production under perfect competition

Market structure
- Efficient alocation and perfect competition
- Monopoly and price discrimination

- Strategic decision making and game theory
- Monopolistic competition and oligopoly
- Economic efficiency and interventions

General equilibrium theory
Marketr failures

- Externalities
- public goods
- asymmetric informations

Course name: **MACROECONOMIC ANALYSIS**
Number of ECTS credits: **6**

**Content:**

- Production function analysis, static analysis of GDP
- Comparative statics in GDP model, dynamic analysis of GDP (discrete and continuous model)
- IS-LM analysis, matrix algebra of IS-LM model
- Mathematics of growth models, Harrod and Domar model
- Solow model of growth, convergence
- Adaptive expectations, rational expectations (Fair-Taylor algorithm)
- Solving models with rational expectations (method of undetermined coefficients)
- Unemployment
- Neoclassical growth model
- Keynesian business cycle theory
- Friedman – Lucas model
- Real business cycle theory
- Phillips curve
- Fiscal and monetary policy
- Quantitative theory of money, Cambridge money demand equation and price level stability
- Economic growth, money and inflation

Course name: **FINANCE**
Number of ECTS credits: **6**

**Content:**

- Finance, Definitions, Corporate Finance, Financial Environment.
- Corporate Financing and Respective Mathematical Models.
- Risk and Return.
- The Cost of Capital and Capital Structure and Respective Mathematical Models.
- Banking.
- Trading on virtual markets.
- Working Capital Management and Respective Mathematical Models.

Course name: **COMPUTER SCIENCE II**
Number of ECTS credits: **6**

**Content:**

- Introduction

Introduction to programming languages, concepts of programming languages, Meta-language, Chomski hierarchy, computability, overview of programming language history.

- Lambda calculus

History of λ-calculus, λ-abstraction, definition of λ-calculus, evaluation, substitution, alpha reductions, beta reductions, programming in λ-calculus, Church numbers, recursion, uses of λ-calculus.

- Syntax

Grammars, parsing, parse trees, BNF, grammar definition, operator, priority of operator, asociativity, dangling else, abstract syntax tree, BNF variations.

- Basic structures

Values, basic types, variable declaration, global declaration, local declaration, implementation of variables, symbol tables, name-spaces.

- Functional languages

Mathematical and logic foundations, function expressions, function definition, recursive functions, polymorphism, higher-order functions, examples of functions.

- Imperative languages

Variables, sequential control, structured control, if statement, loops, patterns, function implementation, parameters, activation records, array, functions on arrays.

- Types

Introduction to types, type declaration, products, records, unions, vectors, recursive types, parametrized types, type checking, type inference, examples of use of types.

- Modules

Modules as units of compilation, interface and implementation, separate compilation, language of modules, information hiding, sharing types among modules, functors, examples of module implementations.

- Objects and classes

Introduction to object-oriented languages, object logic, class definition, aggregation, specialization, inheritance, self and super, object initialization, method overloading, dynamic binding, abstract classes, polymorphism, parametrized classes, introspection, exceptions, implementation of classes and objects.

## COMPULSORY COURSES FOR THE 3RD YEAR OF STUDY

Course name: **FINANCIAL MATHEMATICS**
Number of ECTS credits: **6**

**Content:**

- Introduction in portfolio valuation and basic definitions (arbitration, optimality, efficiency and perfect markets, representative agent, current value).
- Multiperiod model (uncertainty, single agent and optimality, equilibrium and Pareto principle).
- Valuation of options in discrete models.
- Approach with dynamic programming (Bellman equations, Markov uncertainty).

Course name: **STOCHASTIC PROCESSES I**
Number of ECTS credits: **6**

**Content:**

Discrete-time Markov chains, classification of states, strong Markov property, hitting probabilities, ergodic properties.

Continuous time Markov chains: definitions, strong Markov property, left and right equations, birth and death processes, branching processes, ergodic properties, applications.

Martingales, optional stopping times, convergence theorems, applications.

Brownian motion: construction of Brownian motion, properties of trajectories, Markov property, the reflection principle, martingales connected with Brownian motion

Poisson processes: abstract definitions, transformations of Poisson processed, excursion theory.

Course name: **GAME THEORY**
Number of ECTS credits: **6**

**Content:**
- The decision problems in strategic situations.
- Basic concepts of game theory: players, moves, income, matrix game with two players.
- Games in normal form: dominating moves, the best answer, Nash balance.
- Important examples of games in normal form: prisoners' dilemma, game of coordination, partnership struggle, Coin game.
- Random decisions: mixed moves, the existence of Nash balance.
- Dynamic games, games in the branched form: strategies, Nash balance, reversible induction, undergames, perfect balance of undergames.
- Important examples of games in a branched form: centipede game, ultimatum game, the game of negotiations, repeated prisoners' dilemma.
- Comparison of decision theory and human decision making: experiments.
- Behavioral game theory.

Course name: **STATISTICS**
Number of ECTS credits: **6**

**Content:**
Sampling:
- The concept of random sampling
- Sampling distribution and standard error
- Examples of sampling and their standard errors
- Stratified sampling and examples of allocations

Parameter estimation:
- The concept of a statistical model
- Parameter space, estimators, sampling distribution
- Maximum likelihood method
- Asymptotic properties of the maximum likelihood method
- Rao-Cramér inequality, optimality of estimates, factorization theorem

Hypothesis testing:
- Problem formulation
- Statistical tests, test size, power of tests
- Examples of statistical tests
- Wilks' Theorem
- Neyman-Pearson lemma, theory of optimality

Linear models:
- Assumptions of linear models and examples
- Parameter estimation
- Gauss-Markov theorem
- Generalizations of linear models
- Applications

Course name: **ECONOMETRICS**
Number of ECTS credits: **6**

**Content:**
- Introduction and review of probability and statistics
- Linear model, assumptions and BLUE
- Linear regression with one regressor
- Linear regression with multiple regressors
    - Multicolinearity
    - Heteroskedasticity
    - Autocorrelation
- Dummy variables
- Lagged variables

- Nonlinear regression functions
- Maximum likelihood estimator
- Regression with panel data
- Regression with a binary dependent variable
- Instrumental variable regression
- Nonrandom sampling
- Introduction to time series regression and ARMA processes
- Assessing studies and application of econometric techniques on real data

Course name: **FUNDAMENTALS OF INSURANCE**
Number of ECTS credits: **6**

**Content:**
Interest rates
- Definitions and basic formulae.
- Present value of cash-flows.
- Expected present value of cash-flows.

Life insurance products
- Life policies.
- Survival models.
- Expected present value of a policy.
- Net premiums.
- Net premium reserves.
- Collective model.

Non-life insurance
- Non-life policies.
- Claim models.
- Premium calculations
- Reserving.
- Panjer recursion.
- Lundberg process.

Reinsurance
- Proportional reinsurance.
- Nonproportional insurance.

Insurance regulation
- Determination of capital.
- Investments.
- Supervision.
- Solvency 2.

Course name: **MODELLING IN MACROECONOMICS**
Number of ECTS credits: **6**

**Content:**
- Time series (Markov chains, stochastic linear difference equations)
- Dynamic programming (logarithmic preferences)
- Euler equations
- Stochastic control problems
- Practical dynamic programming (expected utility, ris kand preferences, business cycle costs)
- Search, matching, and unemployment (McCall's model, career choice, Jovanovic's model)
- Recursive (partial) equilibrium
- Competitive equilibrium with complete markets

Course name: **FINANCIAL TOPICS IN ENGLISH**
Number of ECTS credits: **6**

**Content:**
The most important research topics in the field of finance and mathematics, among others the following areas:
- Microeconomics,
- Macroeconomics,
- Econometrics,
- Financial Mathematics,
- Finance,
- Statistics.

## ELECTIVE COURSES

*(Read the short descriptions of all elective courses of the study programme. In the table Elective courses you will find the list of the elective courses which were offered in the last two years.)*

Course name: **STOCHASTIC PROCESSES II**
Number of ECTS credits: **6**

**Content:**
- Stochastic integral, Itô's Lemma
- Girsanov Theorem
- Stochastic differential equations
- Stochastic optimal control
- Applications in mathematical finance

Course name: **OPERATIONS RESEARCH**
Number of ECTS credits: **6**

**Content:**
Introduction
- The variety of problems: discrete and continuous optimization, linear and nonlinear optimization, optimization with constraints
- Motivating examples: filling a backpack, traveling salesman, maximum flow, balance and stationary points

Theory
- Linear programming
- Integer programming
- Nonlinear optimization
- Stochastic optimization
- Markov chains and theory of classification

Use
- Business decisions and financial planning
- Logistics: transportation, supply, distribution, storage, inventory
- Optimization of networks

Course name: **RISK MANAGEMENT**
Number of ECTS credits: **6**

**Content:**
The course was prepared under the project “Development of quality and excellence culture of the UP”. It was specifically created for students of social and humanistic study programmes, who wish to obtain knowledge on natural sciences.

1. Introduction.
   - The role of insurance companies.
   - Regulatory framework.
   - Definition of risk and the role od risk management.
2. Reserving in life insurance.
   - Typical life contracts.
   - Discounting and equivalence principle.
   - Survival models.
   - Net premiums.
   - Net reserves.
   - Expense loadings.
   - Collective models.
3. Reserving in non-life insurance.
   - Typical non-life contracts.
   - Statistical bases.
   - Pricing.
   - Deterministic models for claim reserves.
   - Stochastic modeling of aggregate claims.
   - Expense loadings.
4. Risk management and mitigation.
   - Capital requirements under Solvency 2.
   - Matching assets and liabilities.
   - Reinsurance.

Course name: **EU ECONOMIC TRENDS**
Number of ECTS credits: **6**

**Content:**
Content description:

1. Introduction: About E(M)U
   a. Origin (who, when, why)
   b. EU economy in the world
   c. Is there a problem of intergovernmentalism
   d. Fiscal Union: is it feasible?
2. EU growth drivers
   a. Human capital and migration
   b. External balance as a driver for economic growth
   c. Building up a Capital markets union
   d. Sovereign debt and rating agencies
   e. EU TFP
   f. RD, technology and innovation (growth or job disruption?)
   g. The burden of public finance
3. EU challenges
   a. Youth unemployment and an ageing world
   b. Ethics and Islamic finances
4. EU strategies
   a. Brexit and its consequences
   b. EU enlargement to SEE
   c. Governance and the role of supranational EU: a strategic game play