Stable Kneser graphs: problems and conjectures.

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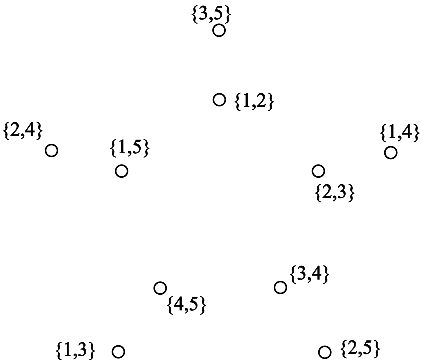

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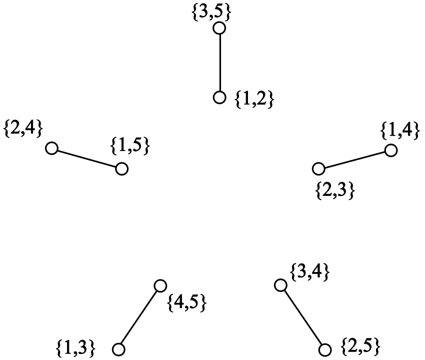

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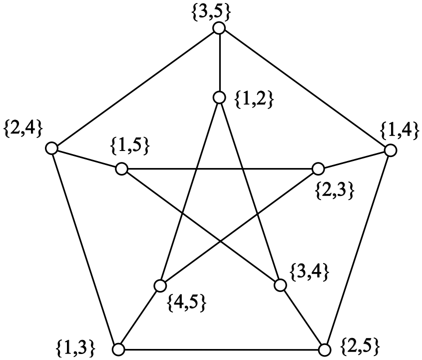

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- S stable set without center $|S| \leq \binom{n-1}{k-1} - \binom{n-k-1}{k-1} + 1$ [*Hilton, Milner (1967)*].

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Are Kneser graphs χ-critical?

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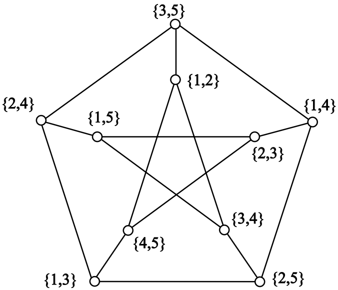

Figure: $KG(5,2)$

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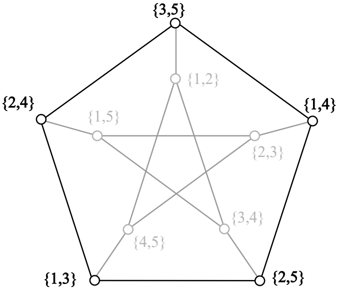

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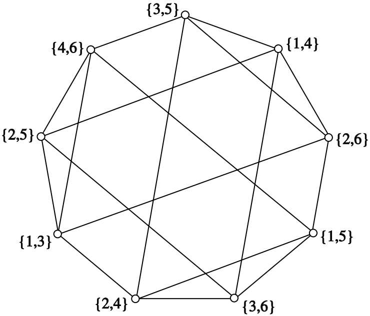

Figure: $KG(6,2)_2$

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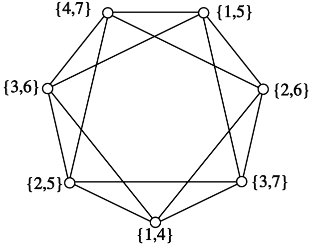

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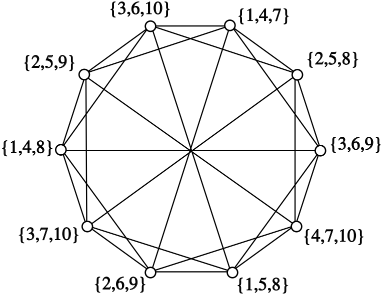

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- $n = 2s+2,\ k = 2$ ✓ $(\chi(KG(2s+2,2)_s) = s+2)$[M. Valencia, T., 2017]
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- $\chi(G) = \min\{k : G \to K_k\}$.

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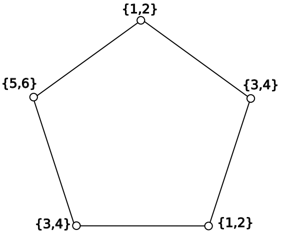

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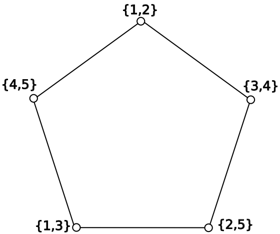

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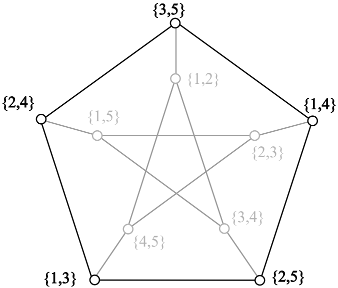

Figure: $KG(5,2)_2$

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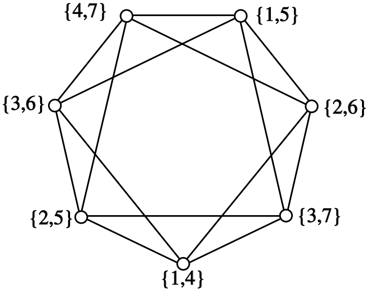

Figure: $KG(7,2)_3$

HOM-IDEMPOTENCE OF STABLE KNESER GRAPHS

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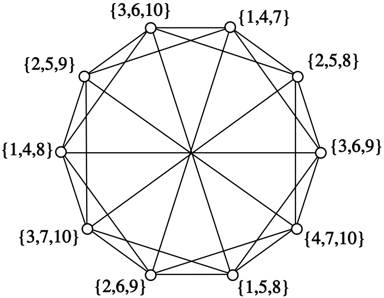

FIGURE: $KG(10,3)_3$

Automorphism of graphs

A bijective homomorphism from G to G is an automorphism.

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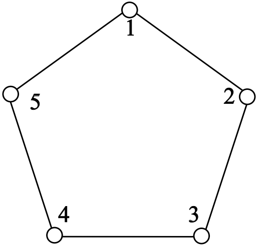

Figure: C_5

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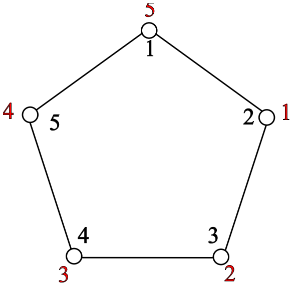

FIGURE: Rotation

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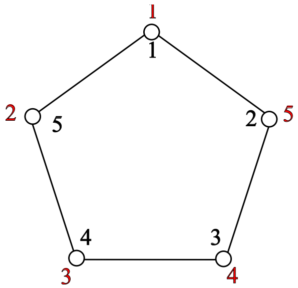

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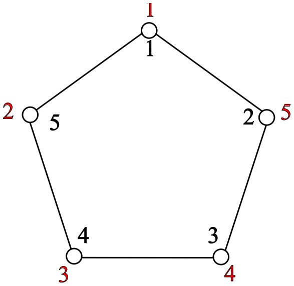

Figure: Reflexion

The automorphism group of C_n, $Aut(C_n)$, is $\mathscr{D}_{2n}$.

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ϕ rotation in $[n]$.

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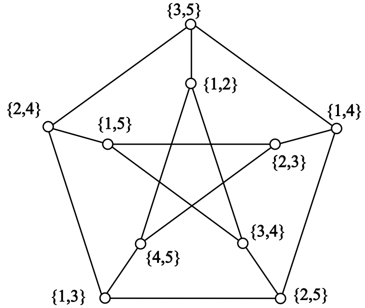

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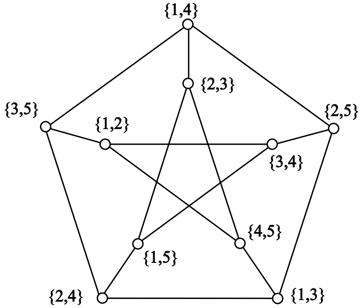

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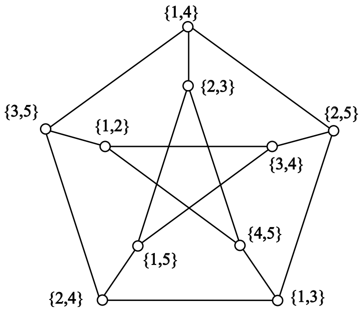

$Aut(KG(n,k))$ is isomorphic to $Sym(n)$.

THE AUTOMORPHISM GROUP OF s-STABLE KNESER GRAPHS

- The automorphism group of the Schrijver graph $Aut(KG(n,k)_2)$ is isomorphic to $\mathscr{D}_{2n}$ [B. Braun, 2010].

The automorphism group of s-stable Kneser graphs

- The automorphism group of the Schrijver graph $Aut(KG(n,k)_2)$ is isomorphic to $\mathscr{D}_{2n}$ [B. Braun, 2010].
- The automorphism group of the stable Kneser graph $Aut(KG(n,k)_s)$ is isomorphic to $\mathscr{D}_{2n}$ [T., 2017].

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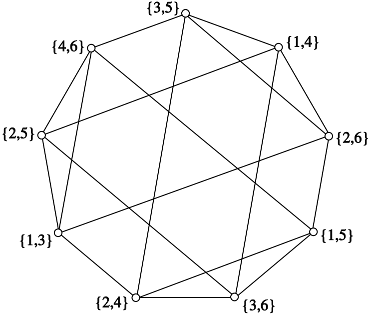

Figure: $KG(6,2)_2$

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- $KG(n,k)_s$ is vertex transitive if and only if $n = sk+1$ [T., 2017].

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- $n = sk+1$, exactly one gap is $s+1$ and the remaining gaps are equal to s.

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- Let $n \geq (k+1)s-1$. Then, the only $2(s-1)$ shifts of the s-stable Kneser graph $\mathrm{KG}(n,k)_{s-\mathrm{stab}}$ are the rotations σ^i with $i \in \{1, \ldots, s-1\} \cup \{n-s+1, \ldots, n-1\}$.

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- The only two shifts of the 2-stable Kneser graph $KG(n,k)_2$ are the rotations σ^1 and σ^{n-1}.

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- Let A be a group and S a subset of A that is closed under inverses and does not contain the identity. The Cayley gragh $Cay(A,S)$ is the graph whose vertex set is A, two vertices u,v being joined by an edge if $u^{-1}v \in S$.

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- Given a graph G, the set of all shifts of G is denoted by S_G.

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- Let G be a χ-critical graph. Then G is weakly hom-idempotent if and only if it is hom-idempotent [Larose et al. 1998].

PROOFS

- A graph G is hom-idempotent if and only if $G \leftrightarrow$ Cay$(\text{Aut}(G^\bullet), S_{G^\bullet})$ [Larose et al. 1998].
- Let G be a χ-critical graph. Then G is weakly hom-idempotent if and only if it is hom-idempotent [Larose et al. 1998].
- Let $n \geq 2k+2$ and let G denotes the graph $KG(n,k)_{2-\text{stab}}$. Then, $G \not\to$ Cay$(\text{Aut}(G), S_G)$, where S_G are the shifts of G.

PROOFS

- A graph G is hom-idempotent if and only if $G \leftrightarrow \text{Cay}(\text{Aut}(G^{\bullet}), S_{G^{\bullet}})$ [Larose et al. 1998].
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- Let $n \geq 2k+2$ and let G denotes the graph $KG(n,k)_{2-\text{stab}}$. Then, $G \not\to \text{Cay}(\text{Aut}(G), S_G)$, where S_G are the shifts of G.
- For any $n \geq 2k+2$, the 2-stable Kneser graphs $KG(n,k)_{2-\text{stab}}$ are not weakly hom-idempotent.

Hvala!

¡Muchas gracias!

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G	r	$\chi_r(G)$	$\chi_r(G\Box G) =$	$\chi_r(G\Box G) \geq$	$\chi_r(G\Box G) \leq$
$KG(5,2)$	2	5	6	-	-
-	3	8	9	-	-
-	4	10	12	-	-
-	5	13	15	-	-
-	6	15	18	-	-
-	7	18	?	20	21
$KG(6,2)$	2	6	8	-	-
-	3	10	12	-	-
-	4	12	?	15	16
-	5	16	?	19	20
-	6	18	?	23	24
$KG(7,2)$	2	7	?	9	10
-	3	12	?	13	15
-	4	14	?	17	20
$KG(8,2)$	2	8	?	11	12
-	3	14	?	16	18
-	4	16	?	21	24