Pólya permanent problem: 100 years after

Alexander Guterman

Moscow State University

This is a joint work with Mikhail Budrevich, Gregor Dolinar, Bojan Kuzma and Marko Orel.

Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \mathrm{per\ A} = \sum_{\sigma \in \mathrm{S_n}} \mathrm{a}_{1\sigma(1)} \cdots \mathrm{a}_{\mathrm{n}\sigma(\mathrm{n})}$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \ldots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

Among our results we prove the following theorem:

Theorem 1. *Suppose $n \geq 3$, and let $\mathbb{F}$ be a finite field with* $\operatorname{char} \mathbb{F} \neq 2$. *Then, no bijective map $T : M_n(\mathbb{F}) \to M_n(\mathbb{F})$ satisfies*

$$\mathrm{per\ A} = \det \mathrm{T(A)}.$$

Also we investigate Gibson barriers (the maximal and minimal numbers of non-zero elements) for convertible $(0, 1)$-matrices and solve several related problems.

Our results are illustrated by the number of examples.