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FOURTH INTERNATIONAL CONFERENCE

Algebraic Combinatorics on the Adriatic Coast

UP Famnit UP Pint



University of Primorska

$(AC)^2 - 2009$

**ALGEBRAIC COMBINATORICS
ON THE ADRIATIC COAST IV**

February 5-7, 2009

UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia



ORGANIZED BY:

University of Primorska

Primorska Institute for Natural Science and Technology (UP PINT)

Faculty of Mathematics, Natural Sciences and Information Technologies (UP FAMNIT)

Institute of Mathematics, Physics and Mechanics (IMFM).

ORGANIZING COMMITTEE

Dragan Marušič (chair), Klavdija Kutnar, Vito Vitrih

WELCOME TO $(AC)^2 - 2009$

We thank all of you for coming and wish you a pleasant and mathematically successful meeting in Koper.

The organization of this meeting would not have been possible without financial support from Primorska Institute for Natural Science and Technology (UP PINT) and Faculty of Mathematics, Natural Sciences and Information Technologies (UP FAMNIT) at University of Primorska, and Institute of Mathematics, Physics and Mechanics, Ljubljana (IMFM).

You are also kindly invited to submit a contribution, that will be published in a special issue of the international journal *Ars Mathematica Contemporanea* (<http://amc.imfm.si/index.php/amc>). When submitting a contribution, please select "Special Issue AC^2 2009" for the section of your submission.

Dragan Marušič

Koper, February 5, 2008

INVITED SPEAKERS

Shaofei Du, *Capital Normal University, China.*

Xingui Fang, *Peking University, China.*

YanQuan Feng, *Beijing Jiaotong University, China.*

Alexander A. Ivanov, *Imperial College, England.*

Aleksander Malnič, *University of Ljubljana, University of Primorska, Slovenia.*

Mikhail E. Muzychuk, *Netanya Academic College, Israel.*

Program

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THURSDAY, February 5, 2009:

15:40 – Dragan Marušič

(University of Primorska and University of Ljubljana, Slovenia)

Opening remarks

16:00–18:00 — Discussion

FRIDAY, February 6, 2009:

10:00–11:20 — INVITED TALKS

10:00 – **Aleksander A. Ivanov**

Imperial College, London, England

A 13-dimensional representation of S_4 which appears in the Monster algebra

10:40 – **Aleksander Malnič**

University of Ljubljana, University of Primorska, Slovenia

Semiregular subgroups and blocks of imprimitivity in transitive groups

11:20 – COFFEE BREAK

11:40–12:20 — CONTRIBUTED TALKS

11:40 – **Pablo Spiga**

University of Padova, Italy

Automorphism groups of Cayley graphs on regular 5-groups

12:00 – **Cui Zhang**

University of Primorska, Slovenia

Two sufficient conditions for vertex-transitive Hamilton graphs of prime-power order

12:20–14:20 – LUNCH

14:20–15:00 — INVITED TALK

14:20 – **Shaofei Du**

Capital Normal University, China

Recent Developments in Regular Maps

15:00–16:20 — CONTRIBUTED TALKS

15:00 – **Jin-Xin Zhou**

Beijing Jiaotong University, China

s-regular cubic Cayley graphs on generalized dihedral groups

15:20 – COFFEE BREAK

15:40 – **Dušanka Janežič**

National Institute of Chemistry, Slovenia

New algorithm for the maximum clique problem

16:00 – **Roman Nedela**

Math. Inst. of Slovak Acad. of Sciences, Slovakia

Regular embeddings of graphs into surfaces

16:20–18:00 — **Discussion**

19:00 – CONFERENCE DINNER

SATURDAY, February 7, 2009:

10:00–11:20 — INVITED TALKS

10:00 – **Mikhail E. Muzychuk**

Netanya Academic College, Israel

On transitive permutation groups containing a bi-cycle

10:40 – **YanQuan Feng**

Beijing Jiaotong University, China

One-regular graphs of square-free order of prime valency

11:20 – COFFEE BREAK

11:40–12:20 — CONTRIBUTED TALKS

11:40 – **Dragan Stevanović**

University of Primorska, Slovenia

Laplacian coefficients and Laplacian-like energy of graphs

12:00 – **István Kovács**

University of Primorska, Slovenia

Automorphism groups of indecomposable S -rings over Z_2^t

12:20–14:20 – LUNCH

14:20–15:00 — INVITED TALK

14:20 – **Xingui Fang**

LAMA and School of Mathematical Sciences, Peking University, China

On Locally Primitive Cayley Graphs of Finite Simple Groups

15:00–16:00 — CONTRIBUTED TALKS

15:00 – **Aleksandar Jurišić**

University of Ljubljana, Slovenia

A characterization of Q -polynomial distance-regular graphs

15:20 – COFFEE BREAK

15:40 – **Janez Žerovnik**

University of Maribor, Slovenia

Multicoloring of hexagonal graphs

16:00–18:00 — Discussion

Abstracts

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Recent Developments in Regular Maps

Shaofei Du

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A 2-cell embedding of a graph into an orientable or nonorientable closed surface is called regular if its automorphism group acts regularly on its arcs and flags respectively. One of central problems in topological graph theory is to classify regular maps by given underlying graphs or automorphism groups. In this talk, we shall present some recent results in regular maps.

On Locally Primitive Cayley Graphs of Finite Simple Groups

Xingui Fang

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In this talk we give some information about locally primitive Cayley graphs of finite groups. First we obtain a sufficient condition under which a locally primitive graph is normal. Then, based on a result of Conder, Li and Praeger, we prove that, except for finite many examples, a locally primitive Cayley graph of a finite nonabelian simple group of valency d is normal if $d \leq 20$ or d is a prime number. Next we construct an infinite family of p -valent non-normal locally primitive Cayley graph of the alternating group for all prime $p > 5$. Finally, we consider locally primitive Cayley graphs of finite simple groups with valency 5 and find all of the candidates of finite nonabelian simple groups G such that the Cayley graph $\text{Cay}(G, S)$ might be non-normal.

One-regular graphs of square-free order of prime valency

Yan-Quan Feng, Yan-Tao Li

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A graph is one-regular if its automorphism group acts regularly on ordered adjacent pairs of the graph. Marušič and Pisanski [Croat. Chemica Acta 73(2000) 969-981] classified one-regular Cayley graphs on a dihedral group of valency 3, and Kwak *et al* [J. Combin. Theory B 98 (2008) 585-598] classified those of valency 5. Recently, the authors classify and enumerate all one-regular Cayley graphs on a dihedral group of any prime valency. It is shown that for an odd prime q , there exists a q -valent one-regular Cayley graph on the dihedral group of order $2m$ if and only if $m = q^t p_1^{e_1} p_2^{e_2} \cdots p_s^{e_s}$ with $m \geq 13$ for $q = 3$ and with $m \geq 31$ for $q = 5$, where $t \leq 1$, $s \geq 1$, $e_i \geq 1$ and p_i 's are distinct primes such that $q \mid (p_i - 1)$. There are exactly $(q-1)^{s-1}$ non-isomorphic such graphs for a given order, which are explicitly constructed. Furthermore, it is shown that every q -valent one-regular graph of square-free order is a Cayley graph on a dihedral group, and as a result, all q -valent one-regular graphs of square-free order are classified and enumerated.

A 13-dimensional representation of S_4 which appears in the Monster algebra

Alexander A. Ivanov

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There are some specific features of the action of the Monster group on its minimal module of dimension 196883. It appears that axiomatising these properties leads to an interesting mathematics. This is motivated by a theorem recently proved by S.Sakuma. We are trying to make a modest further step in developing this area.

New algorithm for the maximum clique problem

Dušanka Janežič, Janez Konc

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A new algorithm for finding a maximum clique in an undirected graph is described. An approximate coloring algorithm has been improved and used to provide bounds to the size of the maximum clique in a basic algorithm which finds a maximum clique. This basic algorithm was then extended to include dynamically varying bounds. The resulting algorithm is significantly faster than the comparable algorithm.

A characterization of Q-polynomial distance-regular graphs

Aleksandar Jurišić

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A new characterization of Q-polynomial distance-regular graphs is given. Let G be a distance-regular graph with diameter $d \geq 3$ and let E be a nontrivial minimal idempotent (i.e., $E \neq E_0$) with the corresponding dual eigenvalues $\theta_0^*, \theta_1^*, \dots, \theta_d^*$. Then E is called *three-term recurrent* if there exist numbers β and γ^* such that $\theta_{i-1}^* - \beta\theta_i^* + \theta_{i+1}^* = \gamma^*$ for $1 \leq i \leq d-1$. The minimal idempotent E is called by M. Lang a *tail* whenever the Schur product $E \circ E$ is a linear combination of E_0 , E and at most one other minimal idempotent of G . If the graph G is Q-polynomial with respect to E , then E is always a tail. In the case when G is bipartite Lang gave necessary and sufficient conditions when a tail extends to a Q-polynomial structure. We generalize his result and characterize the Q-polynomial property of G with respect to E with three well known necessary conditions: $\theta_0^* \neq \theta_i^*$ for $1 \leq i \leq d$, and the minimal idempotent E is three-term recurrent and a tail.

Automorphism groups of indecomposable S-rings over Z_{2^t}

István Kovács

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In the talk I give a description of the automorphism groups of indecomposable S-rings over cyclic groups Z_{2^t} . Such rings have been parameterized by so called atomic sequences by the author (see *Seminar Lotharingien de Combinatoire* **51** (2005), Article B51h). The automorphism groups are constructed in terms of the atomic sequences.

Semiregular subgroups and blocks of imprimitivity in transitive groups

Aleksander Malnič

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The following problem is considered: if H is a semiregular abelian subgroup of a transitive permutation group G acting on a finite set X , find conditions for (non) existence of G -invariant partitions of X . Conditions presented in this talk are derived by studying spectral properties of associated G -invariant digraphs. As an essential tool, irreducible complex characters of H are used. Questions of this kind arise naturally when classifying combinatorial objects which enjoy a certain degree of symmetry. Joint work with István Kovács, Dragan Marušič, and Štefko Miklavič.

On transitive permutation groups containing a bi-cycle

Mikhail E. Muzychuk

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An element $g \in G \leq \text{Sym}(X)$ is called a bi-cycle if the group $\langle g \rangle$ has at most two orbits on X . Such permutation groups appears as automorphism group of dihedrants and, more generally, bicirculants. They also appear as monodromy groups of a rational algebraic complex functions with at most two poles. In my talk I'll present the results about imprimitivity systems of such groups (jointly with F.Pakovich). I'll also report about recent progress in the study of 1-regular dihedrants. The latter results are obtained jointly with I. Kovacs and D. Marušič.

Regular embeddings of graphs into surfaces

Roman Nedela

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A 2-cell embedding of a graph into a closed surface is called regular if its automorphism group acts regularly on the set of arcs. In the talk we shall present a survey on the current status of art on the topic. Recent development will be presented. We shall discuss the problem at a general level. Then we shall concentrate to regular Cayley maps, regular embeddings of particular families of graph including complete graphs, complete bipartite graphs and cubes.

Automorphism groups of Cayley graphs on regular 5-groups

Pablo Spiga

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A Cayley graph Γ on a group G is said to be a normal on G if the right regular representation of G is a normal subgroup of $\text{Aut}(\Gamma)$. It was proved by Y.-Q.Feng and M.-Y.Xu that tetravalent Cayley graphs on regular p -groups are normal. In this talk we investigate the case $p = 5$. All the required definitions are given during the talk.

Laplacian coefficients and Laplacian-like energy of graphs

Dragan Stevanović

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Let G be a graph with Laplacian eigenvalues $\mu_1 \geq \mu_2 \geq \dots \mu_n = 0$. The Laplacian-like energy is defined recently as

$$LEL(G) = \sum_{i=1}^n \sqrt{\mu_i},$$

with the goal of sharing the basic properties of graph energy, while overcoming apparent hardness of proving extremal results on graph energy.

We will first show an important relation between the Laplacian-like energy and the coefficients of the characteristic polynomial of Laplacian matrix of G (the Laplacian coefficients). Then we will use an old characterization of the Laplacian coefficients by Kelmans in order to devise a number of graph transformations that act monotonically on Laplacian coefficients and help solving extremal problems on LEL. This type of problems will be illustrated for trees and unicyclic graphs.

Multicoloring of hexagonal graphs

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A basic problem concerning cellular networks is to assign sets of frequencies (colors) to transmitters (vertices) to avoid unacceptable interference [1]. This leads to some interesting problems on multicoloring induced subgraphs of the infinite triangular lattice. An integer $d(v)$ is assigned to each vertex of the triangular lattice and will be called the demand of the vertex v . A hexagonal graph $G(V, E, d)$ is the vertex weighted graph induced on the subset of the triangular lattice of vertices with positive demand. A proper multicoloring of G is a mapping f from $V(G)$ to subsets of integers such that $|f(v)| \geq d(v)$ for any vertex $v \in G$ and $f(v) \cap f(u) = \emptyset$ for any pair of adjacent vertices u and v in the graph G . The minimal cardinality of a proper multicoloring of G , $\chi(G)$, is called the multichromatic number. Another invariant of interest in this context is the (weighted) clique number, $\omega(G)$, defined as follows: The weight of a clique of G is the sum of demands on its vertices and $\omega(G)$ is the maximal clique weight on G . Clearly, $\chi(G) \geq \omega(G)$. Recently, the bound $\chi(G) \leq (4/3)\omega(G) + C$ was independently proved by several authors [5, 6, 13]. All proofs are constructive thus implying the existence of 4/3-approximation algorithms. McDiarmid and Reed [5] also show that it is NP-complete to decide whether $\chi(G) = \omega(G)$. A distributed algorithm which guarantees the $\lceil (4/3)\omega(G) \rceil$ bound is reported by Narayanan and Shende [6, 7]. A framework for studying distributed online assignment in cellular networks was developed in [4]. In particular, competitive ratios of distributed algorithms which utilize information about increasingly larger neighborhoods are addressed. The best competitive ratios for 0-,1-,2- and 4-local algorithms reported are 3, 3/2, 17/12 and 4/3, respectively. An algorithm is k -local if the computation at a vertex

v uses only information about the demands of vertices whose graph distance from v is less than or equal to k . A 2-local algorithm for multicoloring of hexagonal graphs which uses at most $\lceil (4/3)\omega(G) \rceil$ colors is given in [11]. Better bounds can be obtained for triangle-free hexagonal graphs: [3] provides a distributed algorithm with competitive ratio $5/4$, later a distributed algorithm with competitive ratio $6/5$ is given [14], while an inductive proof for $7/6$ ratio is reported in [2], see also [9]. A 2-local algorithm for a subclass of triangle free hexagonal graphs is given in [12]. McDiarmid and Reed conjectured that for triangle free hexagonal graphs the inequality $\chi(G) \leq (9/8)\omega(G) + C$ holds [5]. If a graph is triangle-free, there is no set of three mutually adjacent vertices of positive demand. It is easy to see that the smallest induced odd cycle in this case is of length 9, hence the constant $9/8$, which is the best possible ratio on C_9 : I will give a short outline of some new results, including a short proof of existence of a 7- [3]multicoloring of triangle free graphs, and possibly of a $7/6$ competitive algorithm for general hexagonal graphs [8].

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Two sufficient conditions for vertex-transitive Hamilton graphs of prime-power order

Cui Zhang

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A Hamilton cycle in a graph is a cycle going through all vertices of the graph, and a graph is said to be hamiltonian if it has a Hamilton cycle. In this talk, two sufficient conditions for the existence of a Hamilton cycle in vertex-transitive graphs of prime-power order are given. Using these conditions, two infinite families of hamiltonian vertex-transitive graphs of order a power of two are constructed. This is a joint work with Jiangtao Shi and Wujie Shi.

***s*-regular cubic Cayley graphs on generalized dihedral groups**

Jin-Xin Zhou

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A graph is *s-regular* if its automorphism group acts regularly on its *s*-arcs. Kutnar and Marušič [J. Combin. Theory B (2008), doi: 10.1016/j.jctb.2008.06.001] proved that connected *s*-regular cubic graphs of girth 6 for some positive integer *s*, apart from four exceptional graphs, are normal Cayley graphs on generalized dihedral groups. In this article all connected *s*-regular cubic Cayley graphs on generalized dihedral groups are classified. As an application, except for the Möbius-Kantor graph of order 16 all 1- or 2-regular cubic graphs of girth 6, described by Kutnar and Marušič as either normal Cayley graphs on generalized dihedral groups or covers of some small graphs, are uniformly constructed as normal Cayley graphs on generalized dihedral groups.

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