MATHEMATICS, UNDERGRADUATE STUDY PROGRAMME, FIRST BOLOGNA CYCLE (MA-07)

COURSE DESCRIPTIONS

COMPULSORY COURSES

COMPULSORY COURSES FOR THE 1ST YEAR OF STUDY

Course name: ALGEBRA I - MATRIX CALCULUS
Number of ECTS credits: 6

Content:
- Vectors, analytic geometry in space.

Course name: ALGEBRA II - LINEAR ALGEBRA
Number of ECTS credits: 6

Content:
- Groups, rings, fields. Ring of polynomials.
- Vector space. Subspaces, linear operators. Linear independence. Basis and dimension of vector space.
- Eigenvalues. The characteristic and minimal polynomial.
- Convexity in the vector space.
- Normalized vector spaces as metric spaces. Isometries of R2 and R3.

Course name: ANALYSIS I - THE FOUNDATIONS OF ANALYSIS
Number of ECTS credits: 6

Content:
- Functions of real variables, even and odd functions, periodicity. Limits of functions, left and right limits. Continuity. Continuous functions on closed intervals limited. Bisection method for finding zeros.
- The elementary functions. Cyclometric functions.
Course name: **ANALYSIS II – INFINITESIMAL CALCULUS**  
Number of ECTS credits: **6**

**Content:**
- The logarithm, the number e, and the definition of exponentiation with the real exponent.
- Drawing planar curves.

Course name: **DISCRETE MATHEMATICS II - COMBINATORICS**  
Number of ECTS credits: **6**

**Content:**

Course name: **MATHEMATICAL PRACTICUM I**  
Number of ECTS credits: **6**

**Content:**
- Programs for presentations (eg PowerPoint), spreadsheet (eg Excel)
- Text editors (eg WinEdt, TextPad, Emacs, Auctech, Open Office, ...)
- Introduction to TeX and LaTeX-a (MikTeX, tetex, GSview, Acrobat Reader, ...)
- The basic tools to produce images (pdf, eps), working with the formats of images including images in LaTeX
- Scanning and use of digital cameras.
Course name: **DISCRETE MATHEMATICS I – SET THEORY**

Number of ECTS credits: **6**

**Content:**
- Introduction to mathematical theory, logic, truth tables, mathematical logic.
- Formal Languages.
- Basic concepts of mathematical logic.
- Finite and infinite, countable and uncountable sets.
- Cardinal and ordinal numbers. Peano arithmetic, mathematical induction.
- The system of axioms of set theory NBG and ZFC. Axiom of choice. Zorn’s lemma.
- Introduction to symbolic computation (Mathematica).

Course name: **MATHEMATICAL TOPICS IN ENGLISH I**

Number of ECTS credits: **6**

**Content:**
Lectures are given on the most current research topics in the field of mathematics, which may include the following topics:
- History of the concept of number
- Number theory
- Algebra
- Analysis
- Famous planning tasks
- Overview of the history of computing
- History of Slovenian mathematics
- Historical development of mathematical concepts

Course name: **COMPUTER SCIENCE I**

Number of ECTS credits: **6**

**Content:**
Basic building blocks of a computer program (using the syntax of the programming language Java):

Basic data structures:

Algorithms and problem solving:

Programming languages overview:

Declarations and types:

Abstraction mechanisms:
Course name: **COMPUTER PRACTICUM**  
Number of ECTS credits: **6**

**Content:**
The faculty network and basic usage rules:
- Description of the faculty computer network, login methods, password changing procedure, e-mail and mailing list usage, access to e-materials.
- OS Linux basics:
  - Description of the Linux OS and its Slovenian version – Pingo Linux. BASH shell usage basics.
- Programming language C:
  - The syntax of the C programming language. Usage of programming language C to solve example problems.

**COMPULSORY COURSES FOR THE 2ND YEAR OF STUDY**

Course name: **ALGEBRA III - ABSTRACT ALGEBRA**  
Number of ECTS credits: **6**

**Content:**
- Introduction to number theory, Euclidean algorithm, congruences.
- Groupoids, semigroups and groups. Homomorphisms of groups. Normal subgroups and factor groups. Families of groups. Groups given by generators and relations. Sylow theorems.

Course name: **ANALYSIS III - FUNCTIONS OF MANY VARIABLES**  
Number of ECTS credits: **6**

**Content:**
- Functions of several variables. Continuity, partial differentiability. Differential mapping from \( \mathbb{R}^n \) to \( \mathbb{R}^m \). Jacobian matrix. Chain rule of differentiation.
- Double and multiple integrals. Properties. The conditions on the existence. The introduction of new variables.
- Calculation and application.
- Proper and generalized integrals with parameter. Beta and Gamma functions. Stirling formula.
Course name: **PHYSICS**
Number of ECTS credits: **6**

**Content:**

Course name: **INTRODUCTION TO NUMERICAL CALCULATIONS**
Number of ECTS credits: **6**

**Content:**

Course name: **PROBABILITY**
Number of ECTS credits: **6**

**Content:**
- Basics of combinatorics
- Fundamental Theorem of combinatorics.
- Variations and variations with repetition.
- Combinations and combinations with repetition.
- Permutations and permutations with repetition.
- The binomial formula and generalizations.
- Outcomes and Events
- The sample space, events, definition of probability.
- Calculations with the events.
- Conditional probability and independence.
- Random Variables
- Random variables and their distributions.
- Overview of some discrete distributions.
- Mathematical expectation and variance.
- Continuous random variables.
- Multidimensional distribution
- Definition of multi-dimensional discrete distribution.
- The independence of random variables.
- Covariance, the sum of random variables.
- Conditional distributions and conditional mathematical expectation.
- Multidimensional continuous distributions.
- Generating functions
- Definition and examples.
- The process of diversification.
- Aproximations of distributions
- Convergence of random variables in the distribution.
- The normal distribution approximation of sums of random variables.
- Poisson approximation

Course name: **MATHEMATICAL TOPICS IN ENGLISH II**
Number of ECTS credits: **6**

Content:
- Basic methods of combinatorics: Classification of discrete problems, basic rules of combinatorics, Selections, Inclusion-exclusion principle, generating functions, rook polynomials
- Combinatorics and recursion: Distributions, Polynomial sequences, Descending powers, Stirling number of first and second kind, Lah numbers and antidifferences, Sums, linear recursion
- Theory of discrete probability, experiment, event, conditional probability, independence, Relay experiments, random variables, Mathematical expectation and variance.

Course name: **COMPUTER SCIENCE II**
Number of ECTS credits: **6**

Content:
- Introduction to programming languages, concepts of programming languages, Meta-language, Chomski hierarchy, computability, overview of programming language history.
- Lambda calculus
History of λ-calculus, λ-abstraction, definition of λ-calculus, evaluation, substitution, alpha reductions, beta reductions, programming in λ-calculus, Church numbers, recursion, uses of λ-calculus.
- Syntax
Grammars, parsing, parse trees, BNF, grammar definition, operator, priority of operator, asosciativity, dangling else, abstract syntax tree, BNF variations.
- Basic structures
Values, basic types, variable declaration, global declaration, local declaration, implementation of variables, symbol tables, name-spaces.
- Functional languages
Mathematical and logic foundations, function expressions, function definition, recursive functions, polymorphism, higher-order functions, examples of functions.
- Imperative languages
Variables, sequential control, structured control, if statement,loops, patterns, function implementation, parameters, activation records, array, functions on arrays.
- Types
Introduction to types, type declaration, products, records, unions, vectors, recursive types, parametrized types, type checking, type inference, examples of use of types.
- Modules
Modules as units of compilation, interface and implementation, separate compilation, language of modules, information hiding, sharing types among modules, functors, examples of module implementations.

- Objects and classes
Introduction to object-oriented languages, object logic, class definition, aggregation, specialization, inheritance, self and super, object initialization, method overloading, dynamic binding, abstract classes, polymorphism, parametrized classes, introspection, exceptions, implementation of classes and objects.

**Compulsory courses for the 3rd year of study**

Course name: **MATHEMATICAL MODELLING**
Number of ECTS credits: **6**

**Content:**
- **Introduction.** What is mathematical modeling? The role of mathematical models in natural sciences and economics. Types of mathematical models.
- **Programming tools.** A short overview of Octave/Scilab.
- **Optimization.** Critical point, minimum, maximum, saddle. Taylor's formula for scalar fields. Local extrema and local extrema under constraints. Newton's method. Applications: discrete catenary, truss stability etc.
- **Calculus of variations.** Standard problem of variation calculus. Isoperimetric problems. Applications: catenary, brachistochrone, truss oscillations, etc.

**Elective courses**

*(Read the short descriptions of all elective courses of the study programme. In the table Elective courses you will find the list of the elective courses which were offered in the last two years.)*

Course name: **ALGEBRAIC GRAPH THEORY**
Number of ECTS credits: **6**

**Content:**
- Eigenvalues of the graph;
- Automorphism group of graph;
- Symmetries of the graph;
- Graphs with transitive automorphism group (vertex-transitive graphs, edge-transitive graphs, arc-transitive graphs, distance-transitive graphs);
Course name: **ALGEBRA IV - ALGEBRAIC STRUCTURES**  
**Number of ECTS credits:** 6

**Content:**
- The extension degree. Tower Theorem. Simple algebraic extension. Splitting field.
- Constructions with ruler and compass. Squaring the circle. Trisecting the angle. Doubling the Cube.
- Constructions of regular polygons.

Course name: **ANALYSIS IV - REAL ANALYSIS**  
**Number of ECTS credits:** 6

**Content:**
- Fourier series. Bessel inequality of vector spaces with inner product.
- Orthonormal system and orthonormalized base. Fourier integral and Fourier transform.
- Differential geometry of curves in the plane and space. The length of the curve. Natural parameter.

Course name: **DIFFERENTIAL EQUATIONS**  
**Number of ECTS credits:** 6

**Content:**
- Bessel differential equation. Solution with the series. Representation with series and integrals.
- Numerical solutions.
- Laplace transform. Inverse formula, properties. Application.

Course name: **FINANCING THE HEALTH SYSTEM**  
**Number of ECTS credits:** 6

**Content:**
- Health.
- Basic definition.
- Indicators of population health.
- Public and private.
- Sources of health care financing.
- The role of co-existence of public and private health care financing.
- Health care systems.
- Bismarck's system of statutory health insurance.
- Beveridge's national health care system.
- The market system of health insurance.
- Classification of health insurance.
- Public statutory health insurance.
- Historical data about the development.
- Content of the statutory health insurance.
- Dilemmas and trends.
- Private health insurance.
- The insurance business.
- Risk factors and premium determination.
- Dilemmas and trends.
- Case studies.
- The growth of health care expenditure and management of growth.
- Offer of private health insurance.
- Absence from work due to illness or injury.
- Health financing and longevity.
- Other current themes.

Course name: **FUNCTIONAL ANALYSIS**
Number of ECTS credits: **6**

**Content:**
- Linear operators and linear functionals. Boundedness of the operator.
- Unbounded operators. Closed operator. Adjoint of densely defined operator.

Course name: **SELECTED TOPICS IN DISCRETE MATHEMATICS**
Number of ECTS credits: **6**

**Content:**
- Association schemes: definition, basic properties, examples, intersection numbers.
- Bose-Mesner algebra: basis, properties.
- Primitive idempotents: definition, Krein parameters.
- Distance-regular graphs: definition, examples, intersection numbers.
- Some necessary conditions for the existence of distance-regular graphs with prescribed intersection numbers.
- Primitive and imprimitive distance-regular graphs.
Course name: SELECTED TOPICS IN COMPUTING METHODS AND APPLICATIONS  
Number of ECTS credits: 6  

Content:  
- Hamiltonian Systems  
- Numerical Integration Methods and Algorithms  
- Lie Formalism  
- Symplectic Integration Methods  
- Numerical Experiments

Course name: SELECTED TOPICS FROM STATISTICS  
Number of ECTS credits: 6

Content:  
- Graphical methods  
- Empirical distribution function.  
- Probabilistic diagrams.  
- Histograms.  
- Models for categorical data  
- Contingency tables.  
- $\chi^2$ tests.  
- Logit and probit models.  
- Time series analysis  
- Time series.  
- Stationary processes.  
- ARMA models.  
- Parameter estimation.  
- Model testing  
- Time series and forecasting

Course name: COMBINATORICS  
Number of ECTS credits: 6

Content:  
- Basic methods of combinatorics: Classification of discrete problems, basic rules of combinatorics, Selections, Inclusion and exclusion principle, generating functions, rook polynomials  
- Combinatorics and recursion: Distributions, Polynomial sequences, Descending powers, Stirling number of first and second kind, Lah numbers and antidifferences, Sums, linear recursion  
- Theory of discrete probability, experiment, event, conditional probability, independence, Relay experiments, random variables, Mathematical expectation and variance.

Course name: COMPLEX ANALYSIS  
Number of ECTS credits: 6

Content:  
- Jensen’s formula. Blaschke products and functions in $H^\infty$.

Course name: **GEOMETRY**  
Number of ECTS credits: **6**

**Content:**  
- Steiner systems  
- Designs  
- Almost linear spaces  
- Linear spaces  
- Configurations, Pappus and Desargues configurations  
- Projective spaces  
- Affine spaces  
- Polar spaces  
- Generalized quadrangles  
- Partial geometries

Course name: **CRYPTOGRAPHY AND COMPUTER SAFETY**  
Number of ECTS credits: **6**

**Content:**  
- Classical ciphers and hoistorical development.  
- Fiestel's cipher and AES (Advanced Encryption Standard).  
- Finite fields and Extended Euclidean algorithm.  
- Public crypto systems, one-way functions and related problems from number theory (testing primality, factorization of integers, discrete logarithm problem)  
- Hash functions and message integrity (authentication)  
- Key exchange protocols and identification protocols  
- Pseudo random number generator  
- Other protocols (flipping a coin over the telephone, mental poker, secret sharing, verification codes, visual cryptography, zero knowledge proofs)  
- Public key infrastructure (PKI), certificate authority (CA)  
- Broader view on cryptography – security of information and network security
Course name: **MATHEMATICAL METHODS IN PHYSICS**
Number of ECTS credits: **6**

**Content:**
  Numerical differentiation, Numerical integration

Course name: **MATHEMATICS: METHODS AND ART**
Number of ECTS credits: **6**

**Content:**
- Generating mathematical truths.
- Mathematics: method and art. Numbers 1, 2, 3, 5, 7 and basic principles of thinking. Real and virtual. Restriction, extension and symmetry. Mathematization of science.
- Mathematics in natural sciences, social sciences, arts and politics. Concrete examples: Parliamentary elections and geometric configurations; Genome, Chinese I-Ching and the hypercube; symmetries of molecular graphs and fullerenes; Sports tournaments and graph manipulation; Albrecht Durer - Melancholy, truncated cube, and Pappus configuration; Durer and magic squares. Primes, factorization, and secret codes.

Course name: **MOLECULAR MODELING**
Number of ECTS credits: **6**

**Content:**
- The concepts of molecular modelling
- Introduction to Classical and Quantum Mechanics
- Potential field and molecular mechanics
- Computer simulation methods
- Molecular dynamics simulations
- Monte Carlo methods
- Using molecular modeling techniques in chemistry, pharmacy, biophysics, etc..

Course name: **OPTIMIZATION METHODS**
Number of ECTS credits: **6**

**Content:**
Basic definitions and examples.
Linear programming.
- Mathematical model.
- Simplex method.
- Application examples from production.
- The theory of duality.
- The transhipment problem.
- Integer linear programming.
Nonlinear programming.
- Extremum of a function from R^n to R.
- Gradient and the Hesse matrix.
- Unconstrained minimization.
- Gradient method.
- Constrained minimization.
- Transformation to the unconstrained problem.
- Karush-Kuhn-Tucker conditions.

Discrete optimization.
- Graphs and digraphs.
- The shortest path problem.
- Breath-first search.
- Dijkstra’s, Prim’s and Kruskal’s algorithm.
- Network flows.
- Ford-Fulkerson’s algorithm.
- Matching and weighted matching problems in bipartite graphs.

Approximation algorithms and heuristics.
- Local optimization.
- 2-approximation algorithm for the vertex cover problem.
- 2-approximation algorithm for the metric traveling salesman problem.
- Christofides algorithm.

Applications on concrete examples of discrete optimization (NP-hard) problems and continuous optimization problems.

Course name: **OPTIMIZATION METHODS IN LOGISTICS**
Number of ECTS credits: **6**

Content:
The basic areas of logistics systems.
Theoretical characteristics of logistics and distribution supply chains
- Material flow.
- Information flow.
- Cash flow.

Major decisions on supply chains.
- Location.
- Production.
- Inventories.
- Transportation.

Linear and nonlinear programming.
Discrete optimization.
Construction algorithms.
The use of heuristics and metaheuristics.
Specific examples of the tasks in logistics and distribution supply chains.
- Warehousing and storage planning.
- Compiling of - preparation of transport units.
- Transportation - Carp (road, rail, ship)

Course name: **INTRODUCTION TO FINANCIAL MATHEMATICS**
Number of ECTS credits: **6**

Content:
Mathematics of life insurance: Interest, the current value, The principle of equivalence, Models of survival, Determination of net premiums., Determination of the net mathematical reserves., Risk management in life insurance.

Course name: **INTRODUCTION TO STATISTICS**
Number of ECTS credits: **6**

**Content:**

**Sampling:**
- The concept of random sampling
- Sampling distribution and standard error
- Examples of sampling and their standard errors
- Stratified sampling and examples of allocations

**Parameter estimation:**
- The concept of a statistical model
- Parameter space, estimators, sampling distribution
- Maximum likelihood method
- Asymptotic properties of the maximum likelihood method
- Rao-Cramér inequality, optimality of estimates, factorization theorem

**Hypothesis testing:**
- Problem formulation
- Statistical tests, test size, power of tests
- Examples of statistical tests
- Wilks' Theorem
- Neyman-Pearson lemma, theory of optimality

**Linear models:**
- Assumptions of linear models and examples
- Parameter estimation
- Gauss-Markov theorem
- Generalizations of linear models

**Applications**

Course name: **SOLVING EQUATIONS: FROM AL-KHWARIZMI TO GALOIS**
Number of ECTS credits: **6**

**Content:**

- Classical algebra and art of solving equations.
- Musa al-Khwarizmi and quadratic equation.
- Renaissance Italy and the formula for the equation of the third and fourth degree.
- Battling with equations.
- Cardano, Ferrari and Fontana - Tartaglia.
- Abel, Galois and the birth of modern algebra.
- Symmetric polynomials.
- Regular pentagon, Regular Heptadecagon.
- Solvability of equations by radicals.
Course name: SYMMETRIC CODES
Number of ECTS credits: 6

Content:
- history of the classical symmetric key encryption schemes
- fundamental concepts in the design of block and stream ciphers,
- modes of operation of symmetric key ciphers,
- cryptographic criteria for encryption schemes,
- security evaluation and generic attacks,
- basic building blocks of symmetric key encryption schemes,
- state-of-art ciphers and their security

Course name: STOCHASTIC PROCESSES
Number of ECTS credits: 6

Content:
- Discrete-time Markov chains, classification of states, strong Markov property, hitting probabilities, ergodic properties.
- Continuous time Markov chains: definitions, strong Markov property, left and right equations, birth and death processes, branching processes, ergodic properties, applications.
- Martingales, optional stopping times, convergence theorems, applications.
- Brownian motion: construction of Brownian motion, properties of trajectories, Markov property, the reflection principle, martingales connected with Brownian motion
- Poisson processes: abstract definitions, transformations of Poisson process, excursion theory.

Course name: GRAPH THEORY
Number of ECTS credits: 6

Content:
- Definitions and basic properties of graphs (paths and cycles in graphs, trees, bipartite graphs).
- Eulerian and Hamiltonian cycles.
- Matchings in graphs (König's theorem).
- Connectivity (Menger's theorem, Mader's theorem).
- Planar graphs (Kuratowski's theorem).
- Graph coloring (Four-Color Theorem, Vizing's theorem).

Course name: GAME THEORY
Number of ECTS credits: 6

Content:
- The decision problems in strategic situations.
- Basic concepts of game theory: players, moves, income, matrix game with two players.
- Games in normal form: dominating moves, the best answer, Nash balance.
- Important examples of games in normal form: prisoners' dilemma, game of coordination, partnership struggle, Coin game.
- Random decisions: mixed moves, the existence of Nash balance.
- Important examples of games in a branched form: centipede game, ultimatum game, the game of negotiations, repeated prisoners' dilemma.
- Comparison of decision theory and human decision making: experiments.

Course name: **CODING THEORY**
Number of ECTS credits: **6**

**Content:**
- mathematical background (groups, rings, ideals, vector spaces, finite fields);
- basic concepts in coding theory;
- algebraic methods for the construction of error correcting codes;
- Hamming codes;
- Linear codes;
- Binary Golay codes;
- Cyclic codes;
- BCH codes;
- Reed-Solomon codes;
- bounds (Hamming, Singleton, Johnson's bound , ...)

Course name: **MEASURE THEORY**
Number of ECTS credits: **6**

**Content:**
- Approximation of a measurable function with continuous function. Lusin's theorem.
- Differentiability of measure, symmetrical derivative of a measure. Absolute continuous functions and fundamental theorem of calculus. Theorem on substitution in integration.
- Product measure and Fubini's theorem. Completion of product Lebesgue measures.

Course name: **NUMBER THEORY**
Number of ECTS credits: **6**

**Content:**
- Prime numbers. Writing numbers in other bases.
- Divisibility criterions. Congruences. Theorems of Fermat and Euler.
- Solving congruence equations. Quadratic reciprocity law.
- Möbius inversion formula.
Course name: **TOPOLOGY**  
Number of ECTS credits: **6**

Content:  
- Connectedness. An ordinary connectedness and connectedness with paths. Components. Local connectedness.  

Course name: **HISTORY AND PHILOSOPHY OF MATHEMATICS**  
Number of ECTS credits: **6**

Content:  
- The history of the concept of number. Main and ordinal numbers in different languages. History of writing numbers: hieroglyphically, alphabetically, transition to positional (Chinese), positional. Algorithms, calculators.  
- A historical overview of computer science (from calculators to computing machines, from calculations to programs, from data to information, between mathematics and engineering).  
- The history of mathematics in Slovenia (textbooks, scientific papers, e.g., Vega)  
- Historical development of mathematical and meta-mathematical terms.

Course name: **PERMUTATION GROUPS**  
Number of ECTS credits: **6**

Content:  
- group action.  
- orbits and stabilizers.  
- extensions to multiply transitive groups.  
- primitivity and imprimitivity.  
- permutation groups and graphs.  
- graph automorphisms, vertex-transitive and Cayley graphs.  
- graphs with a chosen degree of symmetry.  
- permutation groups and designs.
Course name: **SEMINAR – INTRODUCTION TO RESEARCH WORK**
Number of ECTS credits: **6**

**Content:**
The course consists of the most important research topics from the field of mathematics.