MATHEMATICS, UNDERGRADUATE STUDY PROGRAMME, FIRST BOLOGNA CYCLE (MA-07)

COURSE DESCRIPTIONS

COMPULSORY COURSES

COMPULSORY COURSES FOR THE 1ST YEAR OF STUDY

Course name: **ALGEBRA I - MATRIX CALCULUS**
Number of ECTS credits: **6**

Content:
- Vectors, analytic geometry in space.
- Matrices. Types of matrices and basic operations with matrices. Rank of a matrix. Inverse. Systems of linear equations. Matrix interpretation and theorem of solvability. Elementary matrices, Gauss method. Determinants. Cramer's rule.

Course name: **ALGEBRA II - LINEAR ALGEBRA**
Number of ECTS credits: **6**

Content:
- Groups, rings, fields. Ring of polynomials.
- Vector space. Subspaces, linear operators. Linear independence. Basis and dimension of
- vector space.
- Eigenvalues. The characteristic and minimal polynomial.
- Inner product. Orthogonal systems. Gramm-Schmidt process of ortogonalization. Norm. Norm of the matrix and the operator. Normal and related operators.
- Convexity in the vector space.
- Normalized vector spaces as metric spaces. Isometries of R2 and R3.

Course name: **ANALYSIS I - THE FOUNDATIONS OF ANALYSIS**
Number of ECTS credits: **6**

Content:
- The natural numbers. Rational numbers. Real numbers. Complex numbers.
- The sequence of real numbers. Limits and accumulation points. Cauchy condition. Upper and lower limit. Monotone sequences. Bolzano-Weierstrass theorem.
- Series. The convergence criteria. Absolutely and conditionally convergent series.
- Functions of real variables, even and odd functions, periodicity. Limits of functions, left and right limits. Continuity. Continuous functions on closed intervals limited. Bisection method for finding zeros.
- The elementary functions. Cyclometric functions.

Course name: **ANALYSIS II – INFINITESIMAL CALCULUS**
Number of ECTS credits: **6**

Content:
- Derivative. Mean value theorems. Differentiation of monotone functions. L'Hopital's rule. Higher derivatives. Taylor's formula. Local extrema. Convex and concave functions. Inflection points. Tangent method of finding the zeros.
- The indefinite integral. Definite Integrals. Darboux and Riemann sums. Leibniz-Newton formula. Mean value theorems. Integration methods. Applications of the definite integral in geometry. Improper integral. Numerical integration.
- The logarithm, the number e, and the definition of exponentiation with the real exponent.
- Drawing planar curves.
- Sequences and function series. Power series. Taylor series. Elementary
- complex functions.

Course name: **DISCRETE MATHEMATICS II - COMBINATORICS**
Number of ECTS credits: **6**

Content:
- The principle of the sum, product. Counting pairs. Elementary combinatorics. Assignment. Assignment within the set. The existence of a 1-factor. Assignment between two sets, Hall's theorem. König's theorem, applications. Recursion. Generating functions. Linear recursion with constant coefficients. Applications of combinatorics. Inclusion-exclusion principle. Rook polynomial. Möbius inversion. Partially ordered sets and the Möbius function. Theorem on the inversion. Designs. Finite projective planes. Correction code. Steiner systems. Kirkman schoolgirl problem. Ramsey theorem, proof and application. Polya Theory. Burnside's lemma. Polya's theorem.
- Graphs, examples of graphs. Trees. Basic properties, enumeration of trees. The cheapest tree. Operations on graphs. Product of graphs. Deck graphs and voltage graphs. Graphs and groups. Graph automorphism group. Cayley graphs and Frucht theorem. Symmetric graphs. Planarity and duality. Criterion of planarity. Graph embeddings in other plots. Duality and Euler's theorem. Graph coloring. Coloring vertices. Coloring edges. Chromatic polynomial. Directed graphs. Eulerian digraphs. Tournaments. Markov chains. Connectivity. Menger's and Hall's theorem. Different versions of Menger's theorem and Ford-Fulkerson's theorem. Matroid theory. Definitions. Matroids and graphs. Examples of matroids and applications

Course name: **MATHEMATICAL PRACTICUM I**
Number of ECTS credits: **6**

Content:
- Programs for presentations (eg PowerPoint), spreadsheet (eg Excel)
- Text editors (eg WinEdt, TextPad, Emacs, Auctech, Open Office, ...)
- Introduction to TeX and LaTeX-a (MikTeX, tetex, GSview, Acrobat Reader, ...)
- The basic tools to produce images (pdf, eps), working with the formats of images including images in LaTeX
- Scanning and use of digital cameras.

Course name: **DISCRETE MATHEMATICS I – SET THEORY**
Number of ECTS credits: **6**

Content:
- Introduction to mathematical theory, logic, truth tables, mathematical logic.
- Formal Languages.
- Basic concepts of mathematical logic.
- Methods of recording the sets. The basic relations between sets, the basic operations on sets or families of sets. Power set. Relations. Graphs. Equivalence relations. Partial and linear ordering. Latices and Boolean algebra. Well ordering. Function. Special types of functions. Category.
- Finite and infinite, countable and uncountable sets.
- Cardinal and ordinal numbers. Peano arithmetic, mathematical induction.
- The system of axioms of set theory NBG and ZFC. Axiom of choice. Zorn's lemma.
- Introduction to symbolic computation (Mathematica).

Course name: **MATHEMATICAL TOPICS IN ENGLISH I**
Number of ECTS credits: **6**

Content:
Lectures are given on the most current research topics in the field of mathematics, which may include the following topics
- History of the concept of number
- Number theory
- Algebra
- Analysis
- Famous planning tasks
- Overview of the history of computing
- History of Slovenian mathematics
- Historical development of mathematical concepts

Course name: **COMPUTER SCIENCE I**
Number of ECTS credits: **6**

Content:
Basic building blocks of a computer program (using the syntax of the programming language Java):
- Variables, types and expressions. Basic I/O operations. Decision statements. Control structures. Functions and parameters. Programs. Structural decomposition.

Basic data structures:
- Simple types. Arrays. Records. Characters and strings. Data representation in computer memory. Memory allocation. Linked structures. Stack. Queue. List. Tree.

Algorithms and problem solving:
- What is an algorithm? Problem solving strategies. The role of algoirithms in problem solving. Algorithm implementation strategies. Debugging. Recursion – recursive functions, divide-and-conquer principle, backtracking, implementation of recursion.

Programming languages overview:
- Types of programming languages. Flow control. Functions. Subprograms. Namespaces.

Declarations and types:
- Types. Declarations of types. Safe typing. Type checking. Subtypes. Classes. Polymorphism.

Abstraction mechanisms:
- Data abstractions. Simple types. Composite types. Flow abstractions. Subprograms and functions. Abstract data types. Objects and classes. Patterns. Modules.

Course name: **COMPUTER PRACTICUM**
Number of ECTS credits: **6**

Content:
The faculty network and basic usage rules:
- Description of the faculty computer network, login methods, password changing procedure, e-mail and mailing list usage, access to e-materials.
- OS Linux basics:
- Description of the Linux OS and its Slovenian version – Pingo Linux. BASH shell usage basics.
- Programming language C:
- The syntax of the C programming language. Usage of programming language C to solve example problems.

COMPULSORY COURSES FOR THE 2ND YEAR OF STUDY

Course name: **ALGEBRA III - ABSTRACT ALGEBRA**
Number of ECTS credits: **6**

Content:
- Introduction to number theory, Euclidean algorithm, congruences.
- Polynomials o in single variable. Euclidean algorithm. Zeros of polynomials. Solving algebraic equations. Polynomials in several variables. Symmetric polynomials. Fundamental theorem of algebra.
- Grupoids, semigroups and groups. Homomorphisms of groups. Normal subgroups and factor groups. Families of groups. Groups given by generators and relations. Sylow theorems.

Course name: **ANALYSIS III - FUNCTIONS OF MANY VARIABLES**
Number of ECTS credits: **6**

Content:
- Metric spaces. Cauchy-Schwarz inequality. Open and closed sets.
- Compactness and connectedness. Sequences in metric spaces. Cauchy sequences and complete metric spaces. Continuity and uniform continuity. Properties of continuous mappings.
- Functions of several variables. Continuity, partial differentiability. Differential mapping from Rn to Rm. Jacobian matrix. Chain rule of differentiation.
- Higher order partial derivatives. Taylor's formula. Theorem on the locally inverse function and on implicit functions. Local extremal problems, constrained extremal problems.
- Double and multiple integrals. Properties. The conditions on the existence. The introduction of new variables.
- Calculation and application.
- Proper and generalized integrals with parameter. Beta and Gamma functions. Stirling formula.

Course name: **PHYSICS**
Number of ECTS credits: **6**

Content:
Course topics: Physical Measurement, Linear movement, The movement in three dimensions, Forces and motion, Newton's laws, Friction, The kinetic energy and work, Potential energy, energy conservation, A system of particles, The centre of gravity, Momentum, Rotation, The angular momentum, Balance and elastic properties, Gravity, Fluid Mechanics, Oscillation, Waves, General characteristics and types of waves, Sound, Heat, Temperature, The thermodynamic laws, The thermal conductivity, The kinetic theory of gases, Entropy, The electric charge, The electric field, Electric Potential, Capacitance, Electrical resistance, The magnetic field, Induction, Alternating currents and electromagnetic oscillations, Electromagnetic waves, Geometrical optics, Interference and diffraction, Basic concepts of modern physics, Photons and material waves, Material waves, atomic physics, The core of the atom, Special Theory of Relativity.

Course name: **INTRODUCTION TO NUMERICAL CALCULATIONS**
Number of ECTS credits: **6**

Content:
- Fundamentals of numerical computing. The floating point and rounding errors. Calculations in floating points. Stable computational processes and the problem sensitivity. The total error.
- Non-linear equations. Bisection. Tangent method: derivatives, implicit functions, systems
- nonlinear equations. Secant method. Algebraic equations.
- Systems of linear equations. LU decomposition and Cholesky decomposition. Gaussian elimination. Diagonally dominant and tridiagonal matrices. Problem sensitivity. Aposteriori error estimation. Neumann series and iteratively improvement of the accuracy.
- Eigenvalues. Power method, Inverse power method. Schur and Gershogorin theorem.
- Function approximation. Polynomial interpolation. Divided difference. Hermite interpolation.
- Numerical integration. Integration with polynomial interpolation. Composite rules. Gaussian quadrature formulas. Euler-Maclaurin formula
- Numerical solution of ordinary differential equations. Solving differential equations of the first order. The Taylor series method of obtaining solution. Simple methods, the order of the method. Methods of type Runge-Kutta.
- A linear programming. Convexity and linear inequalities. Simplex algorithm.

Course name: **PROBABILTY**
Number of ECTS credits: **6**

Content:
- Basics of combinatorics
- Fundamental Theorem of combinatorics.
- Variations and variations with repetition.
- Combinations and combinations with repetition.
- Permutations and permutations with repetition.
- The binomial formula and generalizations.
- Outcomes and Events
- The sample space, events, definition of probability.
- Calculations with the events.
- Conditional probability and independence.
- Random Variables
- Random variables and their distributions.
- Overview of some discrete distributions.

- Mathematical expectation and variance.
- Continuous random variables.
- Multidimensional distribution
- Definition of multi-dimensional discrete distribution.
- The independence of random variables.
- covariance, the sum of random variables.
- Conditional distributions and conditional mathematical expectation.
- Multidimensional continuous distributions.
- Generating functions
- Definition and examples.
- The process of diversification.
- Aproximations of distributions
- Convergence of random variables in the distribution.
- The normal distribution approximation of sums of random variables.
- Poisson approximation

Course name: **MATHEMATICAL TOPICS IN ENGLISH II**
Number of ECTS credits: **6**

Content:
- Basic methods of combinatorics: Classification of discrete problems, basic rules of combinatorics, Selections, Inclusion-exclusion principle, generating functions, rook polynomials
- Combinatorics and recursion: Distributions, Polynomial sequences, Descending powers, Stirling number of first and second kind, Lah numbers and antidifferences, Sums, linear recursion
- Theory of discrete probability, experiment, event, conditional probability, independence, Relay experiments, random variables, Mathematical expectation and variance.

Course name: **COMPUTER SCIENCE II**
Number of ECTS credits: **6**

Content:
- Introduction

Introduction to programming languages, concepts of programming languages, Meta-language, Chomski hierarchy, computability, overview of programming language history.
- Lambda calculus

History of λ-calculus, λ-abstraction, definition of λ-calculus, evaluation, substitution, alpha reductions, beta reductions, programming in λ-calculus, Church numbers, recursion, uses of λ-calculus.
- Syntax

Grammars, parsing, parse trees, BNF, grammar definition, operator, priority of operator, asociativity, dangling else, abstract syntax tree, BNF variations.
- Basic structures

Values, basic types, variable declaration, global declaration, local declaration, implementation of variables, symbol tables, name-spaces.
- Functional languages

Mathematical and logic foundations, function expressions, function definition, recursive functions, polymorphism, higher-order functions, examples of functions.
- Imperative languages

Variables, sequential control, structured control, if statement,loops, patterns, function implementation, parameters, activation records, array, functions on arrays.
- Types

Introduction to types, type declaration, products, records, unions, vectors, recursive types, parametrized types, type checking, type inference, examples of use of types.

- Modules

Modules as units of compilation, interface and implementation, separate compilation, language of modules, information hiding, sharing types among modules, functors, examples of module implementations.

- Objects and classes

Introduction to object-oriented languages, object logic, class definition, aggregation, specialization, inheritance, self and super, object initialization, method overloading, dynamic binding, abstract classes, polymorphism, parametrized classes, introspection, exceptions, implementation of classes and objects.

COMPULSORY COURSES FOR THE 3RD YEAR OF STUDY

Course name: **MATHEMATICAL MODELLING**
Number of ECTS credits: **6**

Content:

- *Introduction.* What is mathematical modeling? The role of mathematical models in natural sciences and economics. Types of mathematical models.
- *Programming tools*. A short overview of Octave/Scilab.
- *Optimization.* Critical point, minimum, maximum, saddle. Taylor's formula for scalar fields. Local extrema and local extrema under constraints. Newton's method. Applications: discrete catenary, truss stability etc.
- *Calculus of variations.* Standard problem of variation calculus. Isoperimetric problems. Applications: catenary, brachistochrone, truss oscillations, etc.
- *Linear programming*. What is a linear program? Examples of linear programs: optimal diet, flow in a network etc. Forms of linear programs. The fundamental theorem of linear programming. Simplex method. Duality. Integer linear programming and LP relaxation. Applications.
- *Differential equations and systems of differential equations as mathematical models in natural sciences*. Motivational examples. Equilibrium. (Linear) Stability of equilibria. Phase portraits. The basics of Poincare-Bendixon theory. The basics of bifurcation theory. Applications: epidemic models, models of competition, models of symbiosis, predator-prey dynamics, molecular kinetics, basic neurological models, models in economics.

ELECTIVE COURSES

(Read the short descriptions of all elective courses of the study programme. In the table Elective courses you will find the list of the elective courses which were offered in the last two years.)

Course name: **ALGEBRAIC GRAPH THEORY**
Number of ECTS credits: **6**

Content:

- Eigenvalues of the graph;
- Automorphism group of graph;
- Symmetries of the graph;
- Graphs with transitive automorphism group (vertex-transitive graphs, edge-transitive graphs, arc-transitive graphs, distance-transitive graphs);

- Strongly regular graphs.

Course name: **ALGEBRA IV - ALGEBRAIC STRUCTURES**
Number of ECTS credits: **6**

Content:
- Rings. Ideals. Ring homomorphisms. Quotient rings. Integral domains. Euclidean
- rings. Principal ideal domains. Gaussian rings. Gaussian numbers. Chinese remainder theorem.
- Fields. Subfields. Extensions. Finite extensions.
- The extension degree. Tower Theorem. Simple algebraic extension. Splitting field.
- Constructions with ruler and compass. Squaring the circle. Trisecting the angle. Doubling the Cube.
- Constructions of regular polygons.

Course name: **ANALYSIS IV - REAL ANALYSIS**
Number of ECTS credits: **6**

Content:
- Fourier series. Bessel inequality of vector spaces with inner product.
- Orthonormal system and ortnormirana base. Fourier integral and Fourier transform.
- Differential geometry of curves in the plane and space. The length of the curve. Natural parameter.
- Frenet formulas. Surfaces. Curvilinear coordinates, tangentna plane. The first fundamental form.Area of the surface. Surface curvature and second fundamental form.
- Vector analysis. Scalar and vector fields. Gradient, divergence, curl. Potential and solenoid field. Line integrals and surface integrals of the first and second types. Gauss and Stokes theorem.

Course name: **DIFFERENTIAL EQUATIONS**
Number of ECTS credits: **6**

Content:
- Differential Equations. Examples from geometry and physics. Cauchy problem and Euler's method of solution.
- The elementary integration methods for ordinary differential equations. Existence theorems. Differential equations of higher orders. Linear differential equations. Systems of differential equations. Separable variables. Homogeneous right-hand side. Linear equation. Bernoulli and Riccati equatin.
- Calculus of variations. The basic problem of calculus of variations. Euler's equation. Isoperimetric problem.
- Bessel differential equation. Solution with the series. Representation with series and integrals.
- Numerical solutions.
- Laplace transform. Inverse formula, properties. Application.
- Boundary problems for differential equations of second order. Sturm-Liouville operator

Course name: **FINANCING THE HEALTH SYSTEM**
Number of ECTS credits: **6**

Content:
- Health.
- Basic definition.

- Indicators of population health.
- Public and private.
- Sources of health care financing.
- The role of co-existence of public and private health care financing.
- Health care systems.
- Bismarck's system of statutory health insurance.
- Beveridge's national health care system.
- The market system of health insurance.
- Classification of health insurance.
- Public statutory health insurance.
- Historical data about the development.
- Content of the statutory health insurance.
- Dilemmas and trends.
- Private health insurance.
- The insurance business.
- Risk factors and premium determination.
- Dilemmas and trends.
- Case studies.
- The growth of health care expenditure and management of growth.
- Offer of private health insurance.
- Absence from work due to illness or injury.
- Health financing and longevity.
- Other current themes.

Course name: **FUNCTIONAL ANALYSIS**
Number of ECTS credits: **6**

Content:
- Topological vector spaces. Normed spaces. Banach spaces. Finite dimensional normed spaces. Seminorms and local convexity. Minkowsky functional. Closed subspaces and quotient space.
- Linear operators and linear functionals. Boundedness of the operator.
- Baire theorem. Uniform boundedness theorem. Open mapping theorem. Closed graph Theorem on the separation of closed convex sets. Weak and weak - * topology. Banach-Alaoglu theorem.
- Dual. Hahn-Banach theorem. Reflexive spaces. Anihilator of the space. The spectrum of the operator. Arsela-Ascoli theorem. Compact operators. The spectrum of the compact operator
- Hilbert spaces. Orthogonality. Parallelogram identity. Riezs theorem on the representation of the bounded functional. Adjoint operator. Orthonormal bases. Self adjoint, unitary and normal operators.
- Banach algebra. Spectrum. Adjunction of identity. Gelfand-Mazur theorem.
- Unbounded operators. Closed operator. Adjpoint of densely defined operator.

Course name: **SELECTED TOPICS IN DISCRETE MATHEMATICS**
Number of ECTS credits: **6**

Content:
- Association schemes: definition, basic properties, examples, intersection numbers.
- Bose-Mesner algebra: basis, properties.
- Primitive idempotents: definition, Krein parameters.
- Distance-regular graphs: definition, examples, intersection numbers.
- Some necessary conditions for the existence of distance-regular graphs with prescribed intersection numbers.
- Primitive and imprimitive distance-regular graphs.

Course name: **SELECTED TOPICS IN COMPUTING METHODS AND APPLICATIONS**
Number of ECTS credits: **6**

Content:
- Hamiltonian Systems
- Numerical Integration Methods and Algorithms
- Lie Formalism
- Symplectic Integration Methods
- Numerical Experiments

Course name: **SELECTED TOPICS FROM STATISTICS**
Number of ECTS credits: **6**

Content:
- Graphical methods
- Empirical distribution function.
- Probabilistic diagrams.
- Histograms.
- Models for categorical data
- Contingency tables.
- χ^2 tests.
- Logit and probit models.
- Time series analysis
- Time series.
- Stationary processes.
- ARMA models.
- Parameter estimation.
- Model testing
- Time series and forecasting

Course name: **COMBINATORICS**
Number of ECTS credits: **6**

Content:
- Basic methods of combinatorics: Classification of discrete problems, basic rules of combinatorics, Selections, Inclusion and exclusion principle, generating functions, rook polynomials
- Combinatorics and recursion: Distributions, Polynomial sequences, Descending powers, Stirling number of first and second kind, Lah numbers and antidifferences, Sums, linear recursion
- Theory of discrete probability, experiment, event, conditional probability, independence, Relay experiments, random variables, Mathematical expectation and variance.

Course name: **COMPLEX ANALYSIS**
Number of ECTS credits: **6**

Content:
- A complex plane. The extended plane and stereographic projection. Power series with complex arguments. Exponential function. Logarithmic function and root functions.

- Differentiation of complex functions. Cauchy-Riemann equations. Entire functions. Integration of complex functions along the path. Cauchy- theorems. Morera's theorem. Liouville's theorem and the fundamental theorem of algebra. The principle of maximum modulus. Homotopy.
- Isolated singularities. Laurent series. Residues and its applications.
- Harmonic functions. Poisson kernel and Poisson integrals. Solution of Dirichlet problem on the circle. Harnack's theorem. Average value property and harmonic functions. Subharmonic functions.
- Schwarz's Lemma. Principle of maximum modulus. Rado's theorem.
- Approximation of rational functions. Runge's theorem. Conformal mappings. Normal family. Riemann theorem on the conformal equivalence.
- Infinite products. Zeros of holomorphic mappings. Weierstrass factorization theorem. Meromorphic functions and Mittag-Leffler's theorem.
- Jensen's formula. Blaschke products and functions in H^∞.

Course name: **GEOMETRY**
Number of ECTS credits: **6**

Content:
- Steiner systems
- Designs
- Almost linear spaces
- Linear spaces
- Configurations, Pappus and Desargues configurations
- Projective spaces
- Affine spaces
- Polar spaces
- Generalized quadrangles
- Partial geometries

Course name: **CRYPTOGRAPHY AND COMPUTER SAFETY**
Number of ECTS credits: **6**

Content:
- Classical ciphers and hoistorical development.
- Fiestel's cipher and AES (Advanced Encryption Standard).
- Finite fields and Extended Euclidean algorithm.
- Public crypto systems, one-way functions and related problems from number theory (testing primality, factorization of integers, discrete logarithm problem)
- Hash functions and message integrity (authentication)
- Key exchange protocols and identification protocols
- Pseudo random number generator
- Other protocols (flipping a coin over the telephone, mental poker, secret sharing, verification codes, vizual cryptography, zero knowledge proofs)
- Public key infrastructure (PKI), certificate authority (CA)
- Broader view on cryptography – security of information and network security

Course name: **MATHEMATICAL METHODS IN PHYSICS**
Number of ECTS credits: **6**

Content:
- Software: Introduction. Comparison of MATLAB and OCTAVE. Mathematica. Numerical differentiation, Numerical integration
- Ordinary differential equations: differential equations of first order. Radioactive decay. Differential equations of second order. The movement of the projectile. Fluctuations. Movement of the planets.
  Partial differential equations: Laplacova equation. Wave equation. The heat equation.
  Fourirer's types: Fast Fourier Transform. Power spectrum.

Course name: **MATHEMATICS: METHODS AND ART**
Number of ECTS credits: **6**

Content:
- Generating mathematical truths.
- Mathematics: method and art. Numbers 1, 2, 3, 5, 7 and basic principles of thinking. Real and virtual. Restriction, extension and symmetry. Mathematization of science.
- Mathematics in natural sciences, social sciences, arts and politics. Concrete examples: Parliamentary elections and geometric configurations; Genome, Chinese I-Ching and the hypercube; symmetries of molecular graphs and fullerenes; Sports tournaments and graph manipulation; Albrecht Durer - Melancholy, truncated cube, and Pappus configuration; Durer and magic squares. Primes, factorization, and secret codes.

Course name: **MOLECULAR MODELING**
Number of ECTS credits: **6**

Content:
- The concepts of molecular modelling
- Introduction to Classical and Quantum Mechanics
- Potential field and molecular mechanics
- Computer simulation methods
- Molecular dynamics simulations
- Monte Carlo methods
- Using molecular modeling techniques in chemistry, pharmacy, biophysics, etc..

Course name: **OPTIMIZATION METHODS**
Number of ECTS credits: **6**

Content:
Basic definitions and examples.
Linear programming.
- Mathematical model.
- Simplex method.
- Application examples from production.
- The theory of duality.
- The transshipment problem.
- Integer linear programming.

Nonlinear programming.
- Extremum of a function from Rn to R.

- Gradient and the Hesse matrix.
- Unconstrained minimization.
- Gradient method.
- Constrained minimization.
- Transformation to the unconstrained problem.
- Karush-Kuhn-Tucker conditions.

Discrete optimization.

- Graphs and digraphs.
- The shortest path problem.
- Breath-first search.
- Dijkstra's, Prim's and Kruskal's algorithm.
- Network flows.
- Ford-Fulkerson's algorithm.
- Matching and weighted matching problems in bipartite graphs.

Approximation algorithms and heuristics.

- Local optimization.
- 2-approximation algorithm for the vertex cover problem.
- 2-approximation algorithm for the metric traveling salesman problem.
- Christofides algoritem.

Applications on concrete examples of discrete optimization (NP-hard) problems and continuous optimization problems.

Course name: **OPTIMIZATION METHODS IN LOGISTICS**
Number of ECTS credits: **6**

Content:
The basic areas of logistics systems.
Theoretical characteristics of logistics and distribution supply chains

- Material flow.
- Information flow.
- Cash flow.

Major decisions on supply chains.

- Location.
- Production.
- Inventories.
- Transportation.

Linear and nonlinear programming.
Discrete optimization.
Construction algorithms.
The use of heuristics and metaheuristics.
Specific examples of the tasks in logistics and distribution supply chains.

- Warehousing and storage planning.
- Compiling of - preparation of transport units.
- Transportation - Carp (road, rail, ship)

Course name: **INTRODUCTION TO FINANCIAL MATHEMATICS**
Number of ECTS credits: **6**

Content:
Mathematics of life insurance: Interest, the current value, The principle of equivalence, Models of survival, Determination of net premiums., Determination of the net mathematical reserves., Risk management in life insurance.

Market models: Types of securities, Stochastic models of markets, The concept of strategy.
Asset management, Dimensions of risk, An optimal strategy for one period, Dynamic Strategy, CAPM model.
Options: Types of options, The principle of arbitration, The protection and the fundamental theorem of evaluation options, European and American options, Exotic options, Practical aspects of security.
Models of interest rates: The importance of stochastic modeling, Basic designs for current interest rates, Options on interest rates.

Course name: **INTRODUCTION TO STATISTICS**
Number of ECTS credits: **6**

Content:
Sampling:
- The concept of random sampling
- Sampling distribution and standard error
- Examples of sampling and their standard errors
- Stratified sampling and examples of allocations

Parameter estimation:
- The concept of a statistical model
- Parameter space, estimators, sampling distribution
- Maximum likelihood method
- Asymptotic properties of the maximum likelihood method
- Rao-Cramér inequality, optimality of estimates, factorization theorem

Hypothesis testing:
- Problem formulation
- Statistical tests, test size, power of tests
- Examples of statistical tests
- Wilks' Theorem
- Neyman-Pearson lemma, theory of optimality

Linear models:
- Assumptions of linear models and examples
- Parameter estimation
- Gauss-Markov theorem
- Generalizations of linear models

Applications

Course name: **SOLVING EQUATIONS: FROM AL-KHWARIZMI TO GALOIS**
Number of ECTS credits: **6**

Content:
- Classical algebra and art of solving equations.
- Musa al-Khwarizmi and quadratic equation.
- Renaissance Italy and the formula for the equation of the third and fourth degree.
- Battling with equations.
- Cardano, Ferrari and Fontana - Tartaglia.
- Abel, Galois and the birth of modern algebra.
- Basic elements of Galois theory. Automorphisms. Galois extensions. Fundamental theorem of Galois theory.
- Symmetric polynomials.
- Regular pentagon. Regular Heptadecagon.
- Solvability of equations by radicals.

Course name: **SYMMETRIC CODES**
Number of ECTS credits: **6**

Content:
- history of the classical symmetric key encryption schemes
- fundamental concepts in the design of block and stream ciphers,
- modes of operation of symmetric key ciphers,
- cryptographic criteria for encryption schemes,
- security evaluation and generic attacks,
- basic building blocks of symmetric key encryption schemes,
- state-of-art ciphers and their security

Course name: **STOCHASTIC PROCESSES**
Number of ECTS credits: **6**

Content:
- Discrete-time Markov chains, classification of states, strong Markov property, hitting probabilities, ergodic properties.
- Continuous time Markov chains: definitions, strong Markov property, left and right equations, birth and death processes, branching processes, ergodic properties, applications.
- Martingales, optional stopping times, convergence theorems, applications.
- Brownian motion: construction of Brownian motion, properties of trajectories, Markov property, the reflection principle, martingales connected with Brownian motion
- Poisson processes: abstract definitions, transformations of Poisson process, excursion theory.

Course name: **GRAPH THEORY**
Number of ECTS credits: **6**

Content:
- Definitions and basic properties of graphs (paths and cycles in graphs, trees, bipartite graphs).
- Eulerian and Hamiltonian cycles.
- Matchings in graphs (König's theorem).
- Connectivity (Menger's theorem, Mader's theorem).
- Planar graphs (Kuratowski's theorem).
- Graph coloring (Four-Color Theorem, Vizing's theorem).

Course name: **GAME THEORY**
Number of ECTS credits: **6**

Content:
- The decision problems in strategic situations.
- Basic concepts of game theory: players, moves, income, matrix game with two players.
- Games in normal form: dominating moves, the best answer, Nash balance.
- Important examples of games in normal form: prisoners' dilemma, game of coordination, partnership struggle, Coin game.
- Random decisions: mixed moves, the existence of Nash balance.
- Dynamic games, games in the branched form: strategies, Nash balance, reversible induction, undergames, perfect balance of undergames.
- Important examples of games in a branched form: centipede game, ultimatum game, the game of negotiations, repeated prisoners' dilemma.

- Comparison of decision theory and human decision making: experiments.

Course name: **CODING THEORY**
Number of ECTS credits: **6**

Content:

- mathematical background (groups, rings, ideals, vector spaces, finite fields);
- basic concepts in coding theory;
- algebraic methods for the construction of error correcting codes;
- Hamming codes;
- Linear codes;
- Binary Golay codes;
- Cyclic codes;
- BCH codes;
- Reed-Solomon codes;
- bounds (Hamming, Singleton, Johnson's bound , ...)

Course name: **MEASURE THEORY**
Number of ECTS credits: **6**

Content:

- The concept of measureability. σ-algebra of measurable sets. Measurable functions. Borel sets and Borel measurable functions. Measureability of limit functions. Simple functions.
- Integral of nonnegative measurable functions and complex measurable functions. Fatou's lemma. Lebesgue's monotone convergence theorem and Lebesgue's dominated convergence theorem. Sets with measure zero and the concept of equality almost everywhere. Lp spaces.
- Positive Borel measures. Support of a function. Riesz's representation theorem for positive linear functional on algebra of continuous functions with compact support. Regularity of Borelovih measures. Lebesgu's measure.
- Approximation of a measurable function with continuous function. Lusin's theorem.
- Complex measures. Total variation. Absolute continuity. Lebesgue-Radon-Nikodym's theorem. Lp spaces as reflexive Banach spaces.
- Differentiability of measure, symmetrical derivative of a measure. Absolute continuous functions and fundamental theorem of calculus. Theorem on substitution in integration.
- Product measure and Fubini's theorem. Completion of product Lebesgue measures.

Course name: **NUMBER THEORY**
Number of ECTS credits: **6**

Content:

- Divisibility of numbers. Greatest common divisor. Least common multiple. Euclid's algorithm.
- Prime numbers. Writing numbers in other bases.
- Divisibility criterions. Congruences. Theorems of Fermat and Euler.
- Solving congruence equations. Quadratic reciprocity law.
- Linear and quadratic Diophantine equations. Continued fractions. Arithmetical functions.
- Möbius inversion formula.

Course name: **TOPOLOGY**
Number of ECTS credits: **6**

Content:

- Topological spaces. Topological structure on a set. Continuous mappings. Bases and subbases. Separation axioms.
- Compactness. Definition of a compactness. Compact metric spaces. Compact subspaces. Mappings of compact spaces. Locally compact spaces.
- Connectedness. An ordinary connectedness and connectedness with paths. Components. Local connectedness.
- Products. Topological product of finitely many factors. Topological properties of finite products. Topological product of infinitely many factors.
- Real-Valued Continuous Functions. Existence and Extension of Functions. Stone-Weierstrass Theorem.
- Quotient Spaces. Quotient Topology. Mappings of Quotient Spaces. Gluing. Projective Spaces.
- Fundamental Theorems of Topology of Euclidean Spaces. Brouwer Fixed-Point Theorem. Jordan Theorem. Invariant Open Sets. Schönflies Theorem.

Course name: **HISTORY AND PHILOSOPHY OF MATHEMATICS**
Number of ECTS credits: **6**

Content:

- The history of the concept of number. Main and ordinal numbers in different languages. History of writing numbers: hieroglyphically, alphabetically, transition to positional (Chinese), positional. Algorithms, calculators.
- Number Theory - primes, Euclidean algorithm, Diophantine equations. Fractions, rational numbers. Roots, algebraic equations. The symbolism of algebra - unknowns.
- Famous design tasks. Pythagorean Theorem and related content. The number π. Famous curves. Trigonometry. Deductive method in mathematics.
- Rhind and Moscow Papyrus. Babylonian cuneiform script following Neugebauer. Ten classics (Suang-Ching). Euclid's Elements. Archimedes' collected works. Bhaskara: Lilavati. Almagest. Fibonacci: Liber Abaci.
- A historical overview of computer science (from calculators to computing machines, from calculations to programs, from data to information, between mathematics and engineering).
- The history of mathematics in Slovenia (textbooks, scientific papers, e.g., Vega)
- Historical development of mathematical and meta-mathematical terms.

Course name: **PERMUTATION GROUPS**
Number of ECTS credits: **6**

Content:

- group action.
- orbits and stabilizers.
- extensions to multiply transitive groups.
- primitivity and imprimitivity.
- permutation groups and graphs.
- graph automorphisms, vertex-transitive and Cayley graphs.
- graphs with a chosen degree of symmetry.
- permutation groups and designs.

Course name: **SEMINAR – INTRODUCTION TO RESEARCH WORK**
Number of ECTS credits: **6**

Content:
The course consists of the most important research topics from the field of mathematics.