# MATHEMATICS WITH FINANCIAL ENGINEERING, MASTER STUDY PROGRAMME, SECOND BOLOGNA CYCLE

# COURSE DESCRIPTIONS

## BASIC COURSES

Course name: **ASSET PRICING**
Number of ECTS credits: **6**

**Content:**
- *Life insurance mathematics.*
    - interests, present value.
    - Principle of equivalence.
    - Survival models.
    - Net premium determination.
    - Net mathematical reserves.
    - Life insurace risk management.
- *Market types.*
    - Assets.
    - Stochastic markets.
    - Strategy.
- *Asset management.*
    - Risk measurement.
    - One period optimal strategy.
    - Dynamic strategie.
    - CAPM model.
- *Option.*
    - Types of options.
    - Arbitrage.
    - Basic theorem of option pricing.
    - European and american options.
    - Exotic options.
    - Practical considerations of insurance.
- *Interest rates models*
    - Stochastic modelling.
    - Fundamental models for present interest rates.
- *Interest rates options*

Course name: **FINANCIAL MARKETS**
Number of ECTS credits: **6**

**Content:**
- Economic and financial institutions
    - Definition of institutional framework, objective functions and actions
    - Definition of basic institutional concepts (institutions, instruments)
    - Definition of political and economic context and legal constraints.
- Principles of economic analysis
    - Supply and demand, elasticity, price stability
    - Utillity function, consumer behaviour, profit maximisation and the theroy of the firm
- Market structure theory
    - Perfect competition

  - monopoly
  - duopoly (Bertrand, Cournot and Stalckenberg models)
- The theory of the economic growth
  - Production function
  - Stability and economic growth
  - Labour market, investments, capital
- Monetary policy
  - Quantitative theory of money and money market (stable price trajectories and monetary policy)
  - Rational expectations
  - Interest rates (Taylorjevo rule, central banking and objective functions)
  - Price stability (ODE's)
- Fiscal policy
  - Taxes and tax distortions (consumer/producer surplus)
  - Budgetary policy

Course name: **VALUATION OF INSURANCE PRODUCTS**
Number of ECTS credits: **6**

**Content:**
Modeling of risk

- Measures of risk.
- Models of life span, modeling of longevity.
- Models for interest rate risk.
- Agregate risk for short term contracts.
- Lundberg process.
- Models for asset pricess.
- Reinsurance.
- Modeling of risk in non-life insurance.
- Reserving in non'life insurance.

Life insurance

- Life contracts.
- General terms.
- Pricing.
- Reserving.
- Models for surplus distribution.
- Risk management in life insurance.

Course name: **FINANCIAL ENGINEERING PRACTICUM**
Number of ECTS credits: **6**

**Content:**
Classroom work will be based on project work based on selected topics form financial mathematics, insurance mathematics using different programming packages. Some topics covered: option pricing, recursive methods, simulations, »grk« estimations, method of diferences, asset pricing analysis, optimal portfolio problem and it's numerical implementations, Monte-Carlo methods, option pricing on interest rates and exchange rates, simullated trading, estuimating premia with complex insurance products, application of logit and probit models for bankrupcy probability estimation, calculationg mathematical reservations, estimation of long run risk for insurance companies, unit-link products evaluation, evaluation of insurance products with investment risks, optimal investment strategies at given maximal risk.

Course name: **STATISTICS**
Number of ECTS credits: **6**

**Content:**
The course is focused on most important current research areas in mathematical statistics. Among other things, it may include the following topics:

- *Sufficient estimators*
  - Definition of sufficient estimator.
  - Factorization theorem.
    - *Optimality in estimation of parameters*
  - Unbiased estimators.
  - The concept of optimum estimator.
  - Cramér-Rao theorem.
  - Optimum estimators

Course name: **STATISTICAL PRACTICUM**
Number of ECTS credits: **6**

**Content:**

- Overview of the basics in programming language R:

Expressions and objects
Functions and arguments
Vectors and matrices
Factors
Data frames
Computational operations

- The R environment:

Workspace
Printouts
Datasets included with R
Graphics
Programming
Importing data

- Examples of application:

Probabilistic distributions
Descriptive statistics
Testing hypotheses
Regression
Logistic regression
Time series.

Course name: **SELECTED TOPICS IN ALGEBRA**
Number of ECTS credits: **6**

**Content:**
The most important up-to-date research topics in algebra, which among others may include the following contents:

- Linear Algebra,
- Group Theory,
- Ring Theory,
- Field Theory, Galois theory.

Course name: **SELECTED TOPICS OF ANALYSIS**
Number of ECTS credits: **6**

**Content:**
Lectures are given on the most current research topics in the areas of analysis, among others, may include the following topics
- differential manifolds
- Sard's Theorem
- Vector fields on manifolds
- Lie Groups
- differential forms
- Stokes' Theorem

Course name: **SELECTED TOPICS IN DISCRETE MATHEMATICS**
Number of ECTS credits: **6**

**Content:**
The most current research topics in discrete mathematics will be taught, which may include, among others, the following topical subsections:

- Theory of configurations,
- Graph theory,
- Algebraic methods in graph theory,
- Large network theory and analysis,
- Learning on Networks,
- Random graph walks,
- World-Wide Web as a graph.

Course name: **MATHEMATICAL MODELLING**
Number of ECTS credits: **6**

**Content:**
The course is focused on most important current research areas in the theory of mathematical modeling. Among other things, it may include the following topics:

- Optimization (Local minimum, local maximum and saddle point. Taylor's formula for scalar fields. Types of stationary points. Extremes with constraints. Discrete catenary. Newton's method. Method of continuation. Computing the balance of a system of rods.)
- Calcuclus of variations (Standard task of calculus of variations. Isoperimetric problem. Fluctuations of a system of rods. Rotating axis. Shape of a rotating rope.)
- Torsion (Navier's equations. Burden arising from tension. Torsion.)
- Statistics ($\chi^2$ test. Unbiased estimation. Statistical simulations.)
- Combinatorial optimization (Optimization tasks. The assignment problem. The transportation problem. Shortest path in a graph. The maximum flow problem. The traveling salesman problem.)
- Linear programming (Linear program. Artificial feeds. Cutting trunks. Non-standard forms of linear programming. Terminology. Combinatorial nature of linear programming. Simplex method. The initial solution of linear program. The revised simplex method.)
- Cutting trunks (Formulation of the problem. Algorithm. Avoiding the inverse matrix. The knapsack problem.)
- The theory of duality (Definition of duality. Duality theorem. Optimality of the simplex method.)
- Algebraic graph theory (The concept of a graph. Transitive closure. Network. Theorem of subspaces. Cycles and cocycles. Dimensions of the subspaces C and K. Basis in K. Solveing the system $Ax=\chi$. Basis in C.)
- Out of Kilter (The problem. Reduction to the circular flows. Duality. Minty's theorem.)

Course name: **PROBABILITY II**
Number of ECTS credits: **6**

**Content:**
Outcomes, events, σ-algebras
- Sample spaces.
- σ-algebras of events, probability measures.
- Systems of events, Dynkin's lema.
- Independence of events and systems of events.

Distributions as measures
- Distribution as push-forward of measure.
- Discretness, density of distributions.
- Functions of random variables.
- Multivariate distributions, marginal distributions, independence.

Expected value
- Expected value as an abstract integral.
- Expectation as an integral with respect to distribution.
- Variances and covariances.

Conditional expectation
- Conditioning with respect to events and discrete random variables.
- Conditioning with respect to general random variables and σ-fields, existence.
- Properties of conditional expectation.
- Conditional distribution.
- Conditional monotone and dominated convergence theorems.

Transformation of random variables
- Generating functions.
- Characteristic functions, uniqueness theorem.

Convergence of random variables
- Types of convergence, relationships between types of convergence.
- Borel-Cantelli lemmas.
- Laws of large numbers.
- Convergence in distribution.
- Approximation of distributions.

Martingales
- Definitions and properties.
- Optional sampling theorem.
- Convergence of martingales.
- Maximal inequalities.

Course name: **SELECTED TOPICS IN NUMERICAL MATHEMATICS**
Number of ECTS credits: **6**

**Content:**
Basic actual research topics are considered from several fields of numerical mathematics, such as:
- Approximation of functions.
- Numerical analysis of ordinary differential equations
- Numerical analysis of partial differential equations
- Numerical methods for large linear systems and
  Numerical methods for large eigenvalue problems.
- Large scale numerical optimization
- Bezier curves and surfaces

Course name: **STOCHASTIC PROCESSES**
Number of ECTS credits: **6**

**Content:**
Markov processes

- Definition and examples, strong Markov property.
- Transition operators, generators.
- Aditive functionals.
- Potential theory and exponential formulae.

Diffusion processes

- Definitions and examples.
- Characterization of one dimensionel diffusion processes.
- Bessel processes.
- Feynman-Kac formula.
- Connection to partial differential equations.
- Local time and excursion theory.
- Generalization to diffusion processes with jumps.

Lévy processes

- Definitions and examples.
- Markov properties of Lévy processes.
- Subordinators.
- Local time of Lévy processes.
- Fluctuation theory.
- Stable processes.

Processes with positive jumps.

Course name: **GAME THEORY**
Number of ECTS credits: **6**

**Content:**

• The problems of decision making in strategic situations.
• Basic concepts of game theory: players, actions, payoffs, two player matrix games.
• Games in normal form: dominated moves, best response, Nash equilibrium, mixed strategies, the existence of equilibrium, main examples.
• Games in normal form in practice: modeling human decision making.
• Dynamic games, games in extended form: strategies, Nash equilibrium, backwards induction, subgames, subgame perfect equilibrium, main examples.
• Repeated games: infinite repetition, finite repetition, the folk theorem.
• Dynamic games in practice: differences between theory and evidence about human decision-making.
• Decision-making without common knowledge: dynamic games with incomplete information, sequential equilibrium.
• Evolutionary game theory.

## ELECTIVE COURSES

***(Read the short descriptions of all elective courses of the study programme. In the table Elective courses you will find the list of the elective courses which were offered in the last two years.)***

Course name: **TIME SERIES**
Number of ECTS credits: **6**

**Content:**

- Introduction: examples of time series, modeling, transformations of time series.
- Stationary processes: definition, autocorrelation function, forcasting in stationary processes, ARMA models.
- ARMA models: autocorrelation function, parameter estimation, diagnostics, forecasting.
- Nonstationary and seasonal time series, SARIMA models, forecasting for nonstationary time series.
- Multivariate time series: stationarity, multivariatre ARIMA models, estimation, forecasting.
- Introduction to ARCH and GARCH models.

Course name: **ALGEBRAIC COMBINATORICS**
Number of ECTS credits: **6**

**Content:**
The most important up-to-date research topics in algebraic combinatorics, which among others may include the following contents:

- Graph Eigenvalues;
- Automorphism group of a graph;
- Symmetries of a graph;
- Graphs with transitive automorphism group (vertex-transitive graphs, edge-tranzitive graphs, arc-tranzitive graphs, distance-tranyitive graphs);
- Strongly regular graphs and algebraic methods.

Course name: **SELECTED TOPICS IN FUNCTIONAL ANALYSIS**
Number of ECTS credits: **6**

**Content:**
Lectures are emphasized on the most important current research topics in the field of functional analysis, which may include the following subtopics
• topological vector spaces. Generalized sequence
• The weak * compactness
• Operators on Banach and Hilbert space.
• Banach algebra, C * algebras and von Neumann algebras.

Course name: **SELECTED TOPICS OF COMPLEX ANALYSIS**
Number of ECTS credits: **6**

**Content:**
Lectures are emphasized on the most important current research topics in the field of complex analysis, which may include the following subtopics
• holomorphic, harmonic, subharmonic functions.
• holomorphic functions of several variables

Course name: **SELECTED TOPICS IN THEORY OF FINITE GEOMETRIES**
Number of ECTS credits: **6**

**Content:**
The course is focused on most important current areas of research in finite geometries. Among other things, it may include the following topics:

- Steiner systems
- Designs
- Almost linear spaces
- Linear spaces
- Configurations, Pappus and Desargues configurations
- Projective spaces
- Affine spaces
- Polar spaces
- Generalized quadrangles
- Partial geometries

Course name: **SELECTED TOPICS IN NUMBER THEORY**
Number of ECTS credits: **6**

**Content:**
The course is focused on most important current areas of research in number theory. Among other things, it may include the following topics:

- Diophantine equations
- Geometry of numbers
- Additive number theory
- Algebraic number theory

Course name: **SELECTED TOPICS IN TOPOLOGY**
Number of ECTS credits: **6**

**Content:**
Lectures are emphasized on the most important current research topics in the field topology, which may include the following subtopics
• Manifolds, Rimanian manifolds.
• Algebraic topology.

Course name: **FINITE FIELDS THEORY**
Number of ECTS credits: **6**

**Content:**
The most important up-to-date research topics in finite fields theory, which among others may include the following contents:

- The structure of finite fields,
- Polynomials over finite fields,
- Polynomial factorization,
- Equations over finite fields,
- Applications of finite fields.

Course name: **PERMUTATION GROUP THEORY**
Number of ECTS credits: **6**

**Content:**
The most important up-to-date research topics in permutation group theory, which among others may include the following contents:

- Group action.
- Orbits and stabilizers.
- Multiple transitive group actions.
- Primitivity and imprimitivity.
- Permutation groups and graphs.
- Automorphisms of graphs. Tranzitive and Cayley graphs.
- Graphs with a particular degree of symmetry.
- Permutation groups and designs.