MATHEMATICAL SCIENCES (MZ-18)

MASTER STUDY PROGRAMME, SECOND BOLOGNA CYCLE

COURSE DESCRIPTIONS FOR THE

GEOMETRY AND APPLICATIONS STUDY FIELD

BASIC COURSES FOR THE GEOMETRY AND APPLICATIONS CORE

Course name: **SELECTED TOPICS IN ALGEBRA (1)**
Number of ECTS credits: **6**

Content:
Actual research topics are presented from the field of algebra which among others include the following areas:

- linear algebra,
- group theory,
- ring theory,
- field theory, Galois theory.

Course name: **SELECTED TOPICS IN ANALYSIS (1)**
Number of ECTS credits: **6**

Content:
Lectures are given on the most current research topics in the areas of analysis, among others, may include the following topics

- Fourier analysis
- Analysis on manifolds
- Vector analysis. Gauss' and Stokes' theorem.

Course name: **MATHEMATICAL PRACTICUM**
Number of ECTS credits: **6**

Content:
1. Wolfram Mathematica
- elementary calculations, graphs.
- solving standard problems from analysis, linear algebra, differential equations, etc.
- drawing (explicit, implicit, parametric presentation of objects).
- creating interactive and dynamic drawings.
- graphical presentation of NDE and PDE solutions.
- other topics.

2. Matlab
- elementary calculations
- built-in functions
- working with matrices

- writing m functions
- drawing different objects
- solving real problems with Matlab

ELECTIVE COURSES FOR THE GEOMETRY AND APPLICATIONS CORE

Course name: **SELECTED TOPICS IN COMPLEX ANALYSIS**
Number of ECTS credits: **6**

Content:
Lectures are given on the most current research topics in the field of complex analysis, which may include the topics of

- holomorphic, harmonic, subharmonic functions.
- holomorphic functions of several variables

Course name: **SELECTED TOPICS IN TOPOLOGY**
Number of ECTS credits: **6**

Content:
Lectures about the most current research topics in topology, which may include the following content subsections

- manifolds and Riemann manifolds
- Algebraic topology

Course name: **PHILOSOPHY**
Number of ECTS credits: **3**

Content:

- The origins of Western philosophical thought: the beginnings of philosophy in ancient Greece; key characteristics of philosophy and differences between philosophy, religion and science, and between Western, Asian and other reflections on world and man; distinctions between main Western philosophical and cultural traditions. The subject of philosophy.
- *Antiquity*: origins of culture; pre-Socratic philosophy, Socrates, Plato, Aristotle and Hellenic philosophy. The doctrine of being and theory of knowledge.
- *Middle ages, Renaissance and Humanism*: Social and conceptual origins of the middle ages and its historical frame. Nicolas Copernicus, Johannes Kepler, Francis Bacon, Erasmus of Rotterdam and Michel de Montaigne. Historical types of philosophy.
- *Early modern philosophy and Enlightenment*: René Descartes, Thomas Hobbes and David Hume. Philosophical and historical background of enlightenment and its historical consequences: Voltaire, Jean-Jacques Rousseau and Immanuel Kant. Social philosophy.
- *Romanticism and 19th Century*: The role of art and culture; national culture. G.W.F. Hegel, Arthur Schopenhauer; August Comte; Karl Marx; Friedrich Nietzsche. Philosophical axiology and anthropology.
- *Philosophical and cultural currents in 20th Century*: existential phenomenology (Martin Heidegger and Maurice Merleau-Ponty) and eksistencializem (Jean-Paul Sartre); psychoanalysis and surrealism. Logics and analytical philosophy (Ludwig Wittgenstein). The Frankfurt school; hermeneutics, structuralism. Karl Popper and his critics. Philosophy and science.

Course name: **GEOMETRY AND TOPOLOGY**
Number of ECTS credits: **3**

Content:
Geometry and Topology of Manifolds:
i) General Topology (open and closed subsets, connectedness, separability axioms, compactness);
ii) Topological Manifolds, Smooth Manifolds, smooth functions and mappings;
iii) Tangent vector, tangent space, Differential; regular mappings
iv) Local Structure of regular mappings, immersions, embeddings, Whitney Theorem;
v) Orientation and Orientability;
vi) Classification of 2 dimensional closed manifolds;
vii) Tensor Algebra;
viii) Differential forms;
ix) De Rham Cohomologies;
x) Affine Connection, Covariant Derivative, Parallel Transport, Geodesics;
xi) Riemannian Geometry.

Course name: **GEOMETRIC MEASURE THEORY**
Number of ECTS credits: **3**

Content:
- Measures on sigma algebras:
An overview/review of the properties of measurable sets and measures. The completion of a measure/sigma algebra. Borel sigma algebra.
- Caratheodory outer measures, Borel measure, regular measures, Lebesgue measure: Caratheodory's theorem, Lebesgue-Stieltjes measures and increasing right-continuous functions.
- Measurable functions:
An overview/review of the properties of measurable functions.
- Lebesgue integral, Fubini Theorem: An overview/review of the properties of Lebesgue integral of positive/complex functions. Product measure and Fubini theorem. The n-dimensional Lebesgue measure.
- Covering theorems: Vitali and Besicovitch Theorems
- Derivatives of measures: Pointwise derivative of a complex measure with respect to the Lebesgue measure, Functions of bounded variation, Absolutely continuous functions, The Newton–Leibniz formula
- Hausdorff measure and Hausdorff dimension: basic properties, relation between Hausdorff and Lebesgue measure
- Lipschitz mappings: basic properties, relation with Hausdorff measure
- Daniell integral: A construction of measures by means of integrals.

Course name: **GEOMETRIC ASPECTS IN DISCRETE DYNAMICAL SYSTEMS**
Number of ECTS credits: **6**

Content:
Lectures are given on the most current research topics in the field of dynamical systems, which may include the topics of
1. Basic discrete dynamics. Difference equations. The logistic equation. Classification of fixed points. Linearization and Hartman-Grobman theorem. Lyapunov function and Lyapunov exponent. Stable and unstable manifolds.

2. Period doubling and chaos. Hyperbolic systems and Arnold's cat map. Heteroclinic orbits and Smale horseshoe.
3. Polynomial iteration in the complex plane and on the Riemann sphere. Julia, Fatou and Mandelbrot sets. Fatou-Bieberbach domains in C^2.
4. Morse theory, nondegenerate critical points. Gradient flow and topology of level sets. Manifolds as CW complexes. Complex manifolds and CW structure of Stein manifolds.
5. Riemannian manifolds. Connections and geodesics. Curvature tensor, sectional and Ricci curvature. Geodesic flow.

Course name: **GEOMETRICAL OPTIMIZATION PROBLEMS**
Number of ECTS credits: **3**

Content:
(1) Graph Theory. Trees, spanning trees, Kirchhoff Theorem, Minimal Spanning Trees, Kruskal Algorithm.
(2) Reachability Problem and Short Paths Problem, an algebraic approach. Idempotent semi-rings, inductive ordered sets, Fixed Point Theorem, Closed semi-rings, linear equations in semi-rings, applications to graph optimizations problems
(3) Euclidean Minimal Spanning Trees, Delaunay triangulations and Voronoi diagrams.
(4) Shortest trees in the Euclidean plane. Fermat problem. Local structure, Melzak-Weng algorithm. Gilbert-Pollack conjecture and Steiner ratio.
(5) Relations between possible structure of minimal networks and boundary set geometry.
(6) Planar graphs. Pontryagin-Kuratowskii Theorem, Wagner Theorem. Linear embeddings with given angles.

Course name: **SELECTED TOPIS IN DYNAMICAL SYSTEMS**
Number of ECTS credits: **6**

Content:
I. Structural stability of differential equations.
Definition and basic examples, equations on a 2D- torus, Anosov and Grobman-Hartman theorems, Anosov systems, geodesics on the hyperbolic plane.
II. Analytic theory of differential equations. Analiticity of solutions, classification of singular points for linear systems, structure of solution near regular singular points, structure of solutions near irregular singular points, monodromy of differential equations, Stokes graphs and Stokes matrices.
III. Perturbation theory.
Perturbed and unperturbed systems, averaging.
resonances. averaging in one-phase systems, averaging in multi-phase systems, Hamiltonian systems, normal forms and averaging.
IV. Hamiltonian systems.
Linear Hamiltonian systems and quadratic forms, integrable systems, Liouville theorem, geodesic flows, integrable flows on surfaces, Anosov flows.

Course name: **COMBINATORIAL AND CONVEX GEOMETRIES**
Number of ECTS credits: **6**

Content:
Classical results and actual research topics are presented from the field of combinatorial and convex geometry which among others include the following areas

- Convex sets supporting hyperplanes, separation theorems

- Helly's Theorem and its applications
- Facial structure of convex polytopes, cyclic polytopes
- Euler-Poincare formula, regular polytopes
- Sphere packings, density problems
- Theorem of Erdős and Szekeres
- Partitioning of ${\bf R}^d$ by hyperplanes
- Illumination problems, connection to coding theory
- Borsuk's partition problem

Course name: **LIE GROUPS AND LIE ALGEBRAS**
Number of ECTS credits: **3**

Content:
1. Concept of Lie group, main examples. Lie algebras, Lie algebra of a Lie group.
2. Morphisms of Lie groups and induced morphisms of Lie algebras. Subgroups. Cartan Theorem.
3. Actions of Lie groups. Theorem on actions. Corollaries. Orbits and Stabilizers.
4. Representations of Lie groups, induced representations of Lie algebras.
5. Godeman Theorem. Quotients of Lie groups. Corollaries: transitive actions, pre-images of subgroups, intersection of subgroups.
6. First Lie Theorem.
7. One-parametric subgroups. Exponential mapping. Relation to exponential mapping from differential geometry.
8. General properties of connected and simply connected Lie groups. Theorem on simply connected covering Lie group.
9. Second Lie Theorem.
10. Semi-simple Lie algebras.
11. Construction of semi-direct products of Lie groups and Lie algebras.
12. Bott and Gurevich Theorems from algebraic topology. Third Lie Theorem.
13. Classification of Compact Lie groups.

Course name: **COMPUTER AIDED GEOMETRIC DESIGN**
Number of ECTS credits: **3**

Content:
1. Polynomials and splines
2. Bézier curves
3. Bézier surfaces
4. Rational Bézier curves and surfaces
5. B-splines and NURBS
6. Other ways of representing curves and surfaces in Computer aided geometric design.

Course name: **PROJECT MANAGEMENT**
Number of ECTS credits: **3**

Content:
Course content will consist of three sections:
National and EU funding programs of basic and applied research
- Types and methods of financing national and international projects
- Search for calls

- Participation in international networks and partner search
Acquisition, management and implementation of projects
- Project Cycle
- Project management
- Planning the project activities
- Implementation of the project
- Evaluation of the project
From project idea to project
- Project Idea
- General and specific objectives
- Activities
- Project partners
- Plan activities - goals, duration, partners involved, results, milestones, dependencies
- Gantt chart
- Determination of costs
- Consortia agreements of the project partners

Course name: **HISTORY AND METHODOLOGY OF THE SUBJECT**
Number of ECTS credits: **3**

Content:
1. The subject of history and methodology of mathematics and the methods used in it.
- the problem of communication of mathematical knowledge, means of communications (stone engravings, letters, books, papers, blogs, recorded lectures, etc.), problems - solutions. Open problems, conjectures, axioms, definitions, theorems, proofs.
- abstraction, logic, foundation of mathematics
- continuous vs. discrete, two paradigms that drive mathematics.
2. Mathematics in pre-Greek civilizations.
- Egypt, Mesopotamia
3. Mathematics of Ancient Greece.
- Thales, Pythagoras,Euclid's Elements,Archimedes
- Ptolemey, Heron, Diophantus, Pappus
4. Early mathematics outside Europe
-China
-Japan
-Islam
-India
-South America
5. Mathematics in Europe in the Middle Ages and the Renaissance.
- Translations from Arabic into Latin (12h,13h century),The cubic and quartic equations
- Trigonometry, logarithms
6. Mathematics and scientific and technological revolution of the XVI-XVII centuries.
- Descartes, Bernoulli, Huygens,Fermat, Cavlieri
7. The birth of mathematical analysis.
- Newton, Leibniz
8. Development of mathematical analysis in the XVIII century.
- Euler
9. Algebra of the XVIII century.
- Lagrange, Laplace, Vandermonde
10. Mathematics of the XIX century.
- Gauss, Galois, etc.
11. Mathematics of the XIX—XX centuries.

- Lobachevsky,Chebyshev, Riemann, Hilbert, etc.
- Group theory
- Set theory

12. Mathematics in Eastern Europe, Russia and the USSR.

- Important mathematicians that are often overlooked in Western curricula: Bolyai, Lobachevsky, Chebyshev, Alexandrov, Kolmogorov, etc.

13. Mathematics of the XX century.

- Great problems and their solutions, such as four color problem, Fermat's problem, etc.
- Birth and development of selected fields of mathematics, such as topology, combinatorics, theoretical computer science, etc.
- The rise of discrete paradigm to mach the birth of computer and information science, infomation technology, coding and cryptography, understanding of human genome via DNA, computer, traffic and social networks, and logistics.