Adjacency preservers

Marko Orel

Rogla, 18.5.2013

Overview of the talk

1. Preserver problems
2. Adjacency preservers
3. Some techniques related to other mathematical areas
4. Special theory of relativity
5. Hamiltonicity

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Form of a typical preserver problem

M_n = the set of all $n \times n$ matrices

AIM: classification of all maps

$$\Phi : M_n \to M_n$$

that preserve some:

1. function
2. subset
3. relation

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Example 1: Determinant preservers

Frobenius, 1897

A linear bijective map $\Phi : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ preserves determinant, that is, $\det \Phi(A) = \det A$ for all A, if and only if

$$\Phi(A) = PAQ \quad \text{or} \quad \Phi(A) = PA^\top Q,$$

where $\det(PQ) = 1$.

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Example 2: Idempotent preservers

$m = p_1 p_2 \cdots p_k$ product of distinct odd primes,
$\mathbb{Z}_m = \{0, 1, 2, \ldots, m-1\}$

$e \in \mathbb{Z}_m$ is *idempotent* if $e^2 = e$
$A \in M_n(\mathbb{Z}_m)$ is *idempotent* if $A^2 = A$.

A linear map $\Phi : M_n(\mathbb{Z}_m) \to M_n(\mathbb{Z}_m)$ preserves idempotents, that is, $\Phi(A)^2 = \Phi(A)$ whenever $A^2 = A$, if and only if

$$\Phi(A) = eP\big(fA + (1-f)A^\top\big)P^{-1},$$

where $e, f \in \mathbb{Z}_m$ are idempotents.

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Example 3: Adjacency preservers

$\mathcal{M} =$ a set of matrices

$A, B \in \mathcal{M}$ are *adjacent* if $\mathrm{rk}(A - B)$ is minimal and nonzero

$\mathcal{M} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$\mathcal{M} = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

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Adjacency preservers

$\Phi : \mathcal{M} \to \mathcal{M}$ preserves adjacency in both directions, if

$$A, B \text{ are adjacent} \Longrightarrow \Phi(A), \Phi(B) \text{ are adjacent.}$$

$\Gamma = (V, E)$, $V = \mathcal{M}$, $E = \{\{A, B\} \;:\; A \text{ and } B \text{ are adjacent}\}$

Bijective adj. preserves in both directions = automorphisms of Γ

Adjacency preservers = endomorphisms of Γ

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Bijective maps that preserves adjacency in both directions on

$$\mathcal{M} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by Hua's fundamental theorem of geometry of **** matrices.

$\mathcal{M} = H_n(\mathbb{F})$

$$\Phi(A) = \lambda P A^{\sigma} P^{*} + B$$

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(bijectivity and 'both directions' are not assumed)

- $H_n(\mathbb{C})$ (Huang, Šemrl 2008)
- $S_n(\mathbb{R})$ (Legiša 2011)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012)
- $M_{m\times n}(\mathbb{D})$, some additional assumptions (Šemrl, accepted)
- $HGL(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel)

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Some techniques from other math areas

- Graph theory (eigenvalues, chromatic number)
- Geometry ((non)existence of ovoids/spreads in hermitian polar spaces)

Graph Γ is a *core* if $\mathrm{Aut}(\Gamma) = \mathrm{End}(\Gamma)$.

$H_n(\mathbb{F}_{q^2})$
$S_n(\mathbb{F}_q)$, $n \geq 3$
$HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$, $q = 2$

Godsil, Royle, 2011

If Γ connected regular, $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of vertices at distance 2, then Γ is a core or $\chi(\Gamma) = \omega(\Gamma)$

$\chi(\Gamma) \geq 1 + \frac{\lambda_{\max}}{-\lambda_{\min}}$

$H_n(\mathbb{F}_{q^2})$ distance regular
$S_n(\mathbb{F}_q)$ Cayley graph over abelian group

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Special theory of relativity

A Minkowski space-time M_4 is $\mathbb{R}^4$ equipped with a product

$$(\mathbf{r}_1, \mathbf{r}_2) = -x_1 x_2 - y_1 y_2 - z_1 z_2 + c^2 t_1 t_2$$

between events $\mathbf{r}_1 := (x_1, y_1, z_1, ct_1)$ and $\mathbf{r}_2 := (x_2, y_2, z_2, ct_2)$.

A map $\phi : M_4 \to M_4$ preserves the speed of light if

$$\big(\phi(\mathbf{r}_1) - \phi(\mathbf{r}_2), \phi(\mathbf{r}_1) - \phi(\mathbf{r}_2)\big) = 0 \text{ whenever } (\mathbf{r}_1 - \mathbf{r}_2, \mathbf{r}_1 - \mathbf{r}_2) = 0.$$

These maps are closely related to adjacency preservers on 2×2 hermitian matrices.

Problem on hamiltonicity

There are only 5 known connected vertex transitive graphs that are not hamiltonian: K_2, Petersen graph, Coxeter graph, two graphs derived from Petersen/Coxeter graph

$HGL_2(\mathbb{F}_4)$ =Petersen graph
$SGL_3(\mathbb{F}_2)$ =Coxeter graph

$HGL_n(\mathbb{F}_4)$ vertex transitive
$SGL_n(\mathbb{F}_2)$ vertex transitive for odd n

Problem

Are graphs $HGL_n(\mathbb{F}_4)$ and $SGL_m(\mathbb{F}_2)$ hamiltonian for $n \geq 3$ and $m \geq 4$? How to construct a hamiltonian cycle if it exists?

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