### International Conference on Graph Theory and Combinatorics

Dedicated to Prof. Dragan Marušič's 60th Birthday. Koper, Slovenia, May 1 - May 3, 2013,

## TIMETABLE

10:00 A. A. IVANOV: Group Theory and Life
10:25 S. ZHOU: Symmetric Graphs with 2-arc Transitive Quotients
10:50 - - - Coffee - - 11:10 R. NEDELA: Half-arc-transitive Actions with Cyclic Stabilizers and their Geometric Visualisation
11:35 P. SPIGA: Semiregular Elements in Cubic Vertex-transitive Graphs and the Restricted Burnside Problem
11:50 - - Lunch - - 14:30 E. DOBSON: Classifying Vertex-transitive Graphs
14:55 D. WITTE MORRIS: Recent Results on Hamiltonian Cycles in Small Cayley Graphs
15:20 N. SEIFTER: Reachability Relations, Transitive Digraphs and Groups
15:40 - - Coffee - - 16:00 Free discussion
18:00 - - Santomas - - -

Friday, May 3rd
10:00 M. MUZYCHUK: On Regular Cayley Maps over Dihedral Groups
10:25 L. MARTÍNEZ FERNÁNDEZ: *m*-Cayley Circulant Strongly Regular Digraphs
10:50 - - - Coffee - - 11:10 G. KISS: Notes on Semiarcs
11:35 G. VERRET: Semiregular Automorphisms of Arc-transitive Graphs
12:00 - - Lunch - - 16:00 Basketball

# INVITED TALKS

#### **Cayley Graphs On Abelian Groups**

Brian Alspach, *Newcastle University, Australia* Brian.Alspach@newcastle.edu.au

Cayley graphs on abelian groups form the most basic family of Cayley graphs. Nevertheless, they are a source for several nice projects about extending their properties to families of Cayley graphs on other groups. This talk takes a look at that and presents something old, something new, something borrowed and something blue.

#### Half-arc-transitive and Semi-symmetric Graphs

Marston Conder, University of Auckland, New Zealand m.conder@auckland.ac.nz

Nobody could ever accuse Dragan Marušič of 'doing things by halves'. But two of his interests in the study of graph symmetries deal with half-way houses: *half-arc-transitive* graphs (which are vertex- and edge-transitive but not arc-transitive), and *semi-symmetric* graphs (which are edge-transitive but not vertex-transitive). In this talk I will describe the recent discovery of some new examples of these things, inspired by a question posed to me by Dragan at the Bled conference in 1999 (and a follow-up question at the Fields Institute in 2011), and some work I did with Primož Potočnik and Tomaž Pisanski in 2001, leading to our discovery of the *Ljubljana graph*. In particular, as a result of some joint work with Primož Potočnik we now have several examples of half-arc-transitive 4-valent graphs with non-abelian vertex-stabiliser (including at least one with vertex-stabiliser of order 16), and a complete list of all semi-symmetric cubic graphs on up to 10,000 vertices.

#### **Classifying Vertex-transitive Graphs**

Edward T. Dobson, *Mississippi State University, USA, and UP, Slovenia* dobson@math.msstate.edu

We discuss the problem of classifying vertex-transitive graphs. By classifying vertextransitive graphs we mean finding a minimal transitive subgroup of the automorphism group of a graph, typically of smallest order. Thus from our point of view this problem contains the problem of determining whether or not a given graph is isomorphic to a Cayley graph. Our focus will be on classifying all vertex-transitive graphs of a given order.

#### **Group Theory and Life**

Alexander A. Ivanov, *Imperial College London, UK* a.ivanov@ic.ac.uk

#### **Counting Cycles in Vertex-transitive Graphs**

Robert Jajcay, *Comenius University, Slovakia* robert.jajcay@indstate.edu

The easy-to-see fact that the number of cycles of any specified length passing through a fixed vertex of a vertex-transitive graphs is independent of the choice of the vertex has a surprising number of consequences and applications. The aim of this talk is a survey of results based on this observation that happen to be authored by, related to, or influenced by Dragan Marušič. Specifically, we touch on the classification of the orders of non-Cayley vertex-transitive graphs and the poly-circulant conjecture; both problems originated by Dragan. Personal memories involving Dragan and some non-mathematical topics (Grateful Dead?) may not be completely avoidable during the talk.

#### **Notes on Semiarcs**

György Kiss, *Eötvös Loránd University, Hungary* kissgy@cs.elte.hu

Let  $\Pi_q$  be a projective plane of order q. A non-empty pointset  $\mathcal{S}_t \subset \Pi_q$  is called a *t-semiarc* if for every point  $P \in \mathcal{S}_t$  there exist exactly t lines  $\ell_1, \ell_2, \dots, \ell_t$  such that  $\mathcal{S}_t \cap \ell_i = \{P\}$  for  $i = 1, 2, \dots, t$ . These lines are called the tangents to  $\mathcal{S}_t$  at P. The classical examples of semiarcs are the semiovals (t = 1) and the subplanes (t = q - m), where m is the order of the subplane.)

Because of the huge diversity of semiarcs, their complete classification is hopeless. In this talk some characterization theorems about semiarcs having various additional properties will be given.

The talk is based on a joint work with Bence Csajbók (University of Basilicata, Potenza).

#### m-Cayley Circulant Strongly Regular Digraphs

Luis Martínez Fernández, *University of the Basque Country, Spain* luis.martinez@ehu.es

This talk is a survey about strongly regular digraphs admitting a semiregular cyclic group of automorphisms. Some new types of such digraphs, called uniform and almost uniform, are studied. By using partial sum families, the form of the parameters is determined and some directed strongly regular graphs with previously unknown parameters are obtained.

#### **On Regular Cayley Maps over Dihedral Groups**

Mikhail Muzychuk, Netanya Academic College, Israel mikhail@netvision.net.il

In the talk we'll present recent results about G-arc-regular dihedrants. As an application the classification of regular dihedral maps of order 2n, n odd will be presented. This is a joint work with Dragan Marušičc and István Kovács.

#### Half-arc-transitive Actions with Cyclic Stabilizers and their Geometric Visualisation

Roman Nedela, *Matej Bel University, Slovakia* nedela@savbb.sk

During years 1993-1998 I was repeatedly invited by prof. Marušič to visit him at Pedagoška fakulteta in Ljubljana. The starting stay was for one semester, I was giving a course in Elementary number theory. All the teaching duties were organized in one day (Monday) and the rest of week there was a plenty of time to talk with Dragan about mathematics. These discussions were very fruitful and the results of our joint work were published in four papers investigating 4-valent half-arc-transitive graphs from different points of view. The deepest result, in my opinion, is the classification of vertex-stabilisers of such actions published in J. Group Theory. However, the most popular within the community became our first paper relating half-arc-transitive actions with  $Z_2$  stabilisers and regular maps on orientable closed surfaces. In my talk I will explain this correspondence and show some applications and generalisations.

### Reflecting on some Common Work with Dragan (Dedicated to Dragan Marušič at his 60th birthday, May 1, 2013)

Tomaž Pisanski, University of Ljubljana and University of Primorska, Slovenia tomaz.pisanski@fmf.uni-lj.si

In this talk I will reflect on some work I did with Dragan Marušič, mostly during the years 1999 - 2005. I will briefly present the most important results and their implications. I will also give information about the growth of our most successful joint project: the journal Ars Mathematica Contemporanea.

#### **Reachability Relations, Transitive Digraphs and Groups**

Norbert Seifter, *Montanuniversittät Leoben, Austria* seifter@unileoben.ac.at

In [Reachability relations in digraphs, *Europ. J. Combin.* 29 (2008), 1566–1581] certain *reachability relations* on vertex sets of digraphs were introduced and it was shown that some of the properties of digraphs, such as growth, property **Z** and the number of ends, are reflected by the properties of these relations.

In this talk we continue the investigations of these relations with the focus on their connections to properties of the automorphism groups of transitive digraphs. In particular, one of the main results of this paper shows that if a transitive digraph admits a nilpotent subgroup of automorphisms with finitely many orbits, then its nilpotency class and the number of orbits are closely connected to certain properties of the reachability relations on the digraph in question.

The obtained results have interesting implications for Cayley digraphs of certain types of groups such as torsion-free groups of polynomial growth.

Joint work with Aleksander Malnič, Primož Potočnik, Norbert Seifter, Primož Šparl

#### Semiregular Elements in Cubic Vertex-transitive Graphs and the Restricted Burnside Problem

Pablo Spiga, University of Milano-Bicocca, Italy pablo.spiga@unimib.it

One of Dragan's babies is now called "Polycirculant conjecture", that is, every non-trivial vertex-transitive graph admits a non-trivial semiregular automorphism. This beautiful and easy to state conjecture has stimulated very much research in algebraic graph theory and in permutation groups.

With a very nice argument Dragan and Scappellato have shown that every vertex-transitive cubic graph has a semiregular automorphism. In this talk we are interested in the maximal order of such an element. For a cubic Cayley graph or a cubic arc-transitive graph X, with an application of the positive solution of the restricted Burnside problem, we show that the maximal order of a semiregular element in the automorphism group of X tends to infinity as the number of vertices of X tends to in finity.

#### Semiregular Automorphisms of Arc-transitive Graphs

Gabriel Verret, University of Western Australia, Australia, and UP, Slovenia gabriel.verret@fmf.uni-lj.si

A semiregular automorphism of a graph is a nontrivial automorphism, all of whose cycles have the same length. In 1981, Dragan conjectured that every vertex-transitive digraph has a semiregular automorphism. A lot of work has been done on this conjecture but it is still wide open. We discuss some recent results on this conjecture, including a proof of the conjecture for arc-transitive graphs of valency twice a prime.

This is joint work with Michael Giudici.

#### **Recent Results on Hamiltonian Cycles in Small Cayley Graphs**

Dave Witte Morris, *University of Lethbridge, Canada* Dave.Morris@uleth.ca

For more than three decades, Dragan has been a major force in the effort to show that every connected Cayley graph has a hamiltonian cycle. We will discuss two recent developments concerning Cayley graphs on groups that are "small" in an appropriate sense. In one series of papers, the group itself is "small" (meaning that its order has few prime factors). Other papers assume the commutator subgroup is "very small" (meaning that its order has only two prime factors). Dragan participated in the first project, and proposed the second problem.

#### Symmetric Graphs with 2-arc Transitive Quotients

Sanming Zhou, University of Melbourne, Australia smzhou@ms.unimelb.edu.au

A graph *X* is *G*-symmetric if *X* admits *G* as a group of automorphisms acting transitively on the set of vertices and on the set of arcs of *X*, where an arc is an ordered pair of adjacent vertices. In the case when *G* is imprimitive on the vertex set V(X) of *X*, namely when V(X)admits a nontrivial *G*-invariant partition  $\mathcal{B}$ , the quotient graph  $X_{\mathcal{B}}$  of *X* with respect to  $\mathcal{B}$ is always *G*-symmetric and sometimes even (*G*,2)-arc transitive. (A *G*-symmetric graph is (*G*,2)-arc transitive if *G* is transitive on the set of oriented paths of length two.) We obtain necessaryconditions for  $X_{\mathcal{B}}$  to be (*G*,2)-arc transitive (regardless of whether *X* is (*G*,2)-arc transitive) in the case when v-k is an odd prime *p*, where *v* is the block size of  $\mathcal{B}$  and *k* is the number of vertices in a block having neighbours in a fixed adjacent block. These conditions are given in terms of *v*, *k* and two other parameters with respect to (*X*,  $\mathcal{B}$ ) together with a certain 2-point transitive block design induced by (*X*,  $\mathcal{B}$ ). We prove further that if *p* = 3 or 5 then these necessary conditions are essentially sufficient for  $X_{\mathcal{B}}$  to be (*G*,2)-arc transitive.

This talk is based on joint work with Guangjun Xu.

### PARTICIPANTS

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